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CONTROL OF A GLIDING PARACHUTE SYSTEM IN A NON-UNIFORM WIND

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This report investigates a method for post deployment guiding of a cargo-carrying gliding parachute to a target. Least squares estimation and an open-loop control law based on geometric considerations are combined to define a closed-loop control law for the system under variable wind conditions. Simulation studies of the overall system are included for a variety of initial conditions and wind profiles. These simulations indicate that the proposed algorithm, with additional experimentation, may be a feasible solution to the problem.

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PREFACE

This report was prepared under contract with Brown University in the Division of Engineering and Lefschetz Center for Dynamical Systems. The work was carried out under Exploratory Development, Project 1F262203AH86, Control of Gliding Parachute Systems, for the U.S. Army Natick Research and Development Command, Natick, Massachusetts. Mr. Arthur L. Murphy, Jr., of the Engineering Sciences Division, Aero-Mechanical Engineering Laboratory, was the Project Engineer for this effort.

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CONTROL OF A GLIDING PARACHUTE SYSTEM IN A NON-UNIFORM WIND

I. INTRODUCTION

The basic philosophy underlying an approach to the control of a gliding parachute system in a non-uniform wind was introduced in Section I of Pearson [1] with continuing investigations reported in [2] and [3]. This philosophy separates the wind and initial heading estimation problems from the control problem in minimizing the terminal distance of the parachute from a known target while orienting the parachute upwind at the terminal time. Various aspects of the control problem were considered in [1-3] including a computer simulation study of a Differential Dynamic Programming algorithm for solving the open-loop optimal control problem [2], a parameter search algorithm and analytical investigation for the optimal control problem [3], and a bang-off-bang control algorithm based on geometric considerations [3].

In this report the wind estimation and initial heading estimation problems are examined in Section II with particular emphasis given to a least squares formulation. Using the bang-off-bang (open-loop) control law described in Section VI of [3], the least squares and open-loop control algorithms are combined to yield a closed-loop control law which has been simulated under a variety of non-uniform wind conditions. The results of this simulation are included in Section III. Other types of estimation schemes have been considered in this study and are discussed in Section II, but only the least squares algorithm has been used in these initial simulations of the closed-loop control law due to the relative simplicity in computing the least squares estimate.

The equations of motion used throughout this study, [1-3], are the kinematic relations for a uniform descent of the gliding parachute system after full

deployment has ensued:

$$\dot{p}_1(t) = a \cos \theta(t) + w_1(t)$$
$$\dot{p}_2(t) = a \sin \theta(t) + w_2(t) \qquad 0 \leq t \leq T \tag{1}$$
$$\dot{\theta}(t) = \frac{g}{a} \tan \phi(t) \; .$$

In these relations, $(p_1(t), p_2(t))$ denote the position coordinates at time t of the parachute in the horizontal plane relative to the target, $(w_1(t), w_2(t))$ denote the velocity components of the wind vector (assumed to lie in the horizontal plane at all times), $\theta(t)$ is the instantaneous heading of the parachute velocity vector relative to fixed coordinates, and $\phi(t)$ is the parachute bank angle relative to the local vertical. The magnitude of the parachute velocity vector relative to the wind is denoted by "a" in Eq. (1), a presumed known constant of sufficient magnitude to facilitate a wind penetration capability; T is the time to go until touchdown from the initial launch time zero. Alternatively, the third equation in (1) can be expressed in terms of the instantaneous radius of turn of the parachute, $r(t)$, in the horizontal plane via the well known kinematic relation

i.e.,

$$\tan \phi = \frac{a^2}{gr} \tag{2}$$

$$\dot{\theta}(t) = \frac{a}{r(t)} \; . \tag{3}$$

Let the time interval $0 \leq t \leq T$ be divided into N non-overlapping subintervals $t_i \leq t \leq t_{i+1}$, $i = 0,1...N-1$, with $t_o = 0$ and $t_N = T$. The estimation problem relative to the i-<u>th</u> subinterval, $t_i \leq t \leq t_{i+1}$, consists of estimating the initial heading, $\theta(t_i)$, and the wind profile $w(t)$ over $t_i \leq t \leq T$, based on observed data collected over the previous subinterval or intervals. The observed data is assumed to be comprised of the parachute bank angle $\phi(t)$, the position vector $p(t)$, and possibly (depending on the estimation scheme) the total velocity vector of the parachute $\dot{p}(t)$. Given the estimates $\hat{\theta}(t_i)$ and $\hat{w}(t)$ for $t_i \leq t \leq T$, the control problem relative to the i-<u>th</u> subinterval consists of choosing the bank angle $\phi(t)$, or equivalently the turning radius $r(t)$, on $t_i \leq t \leq t_{i+1}$ such

that the parachute would land as close to the target as possible in an upward wind direction at the terminal time if, in fact, the estimates $\hat{\theta}(t_i)$ and $\hat{w}(t)$ were exact and $\phi(t)$ were applied for all t in the interval $t_i \leq t \leq T$. The estimates $(\hat{\theta},\hat{w}(\cdot))$ are updated over the next subinterval based on the new data collected over that interval, and similarly the control variable $\phi(t)$ is re-computed based on the new estimates, resulting in a step-by-step control-estimation sequence which constitutes the closed-loop control algorithm. As discussed in previous reports, control is assumed to be effected through the use of an on-board servo motor attached to the support lines of the gliding parachute with the actual relation between $\phi(t)$ and the angular position of the servo motor to be determined by the particular hardware so assembled. All computations would presumably be performed by a digital computer located at the target with appropriate telecommunications linking the ground based target and the parachute. However, the computations are sufficiently simple that on-board digital computations might be feasible if such were desired.

II. WIND AND INITIAL HEADING ESTIMATION

Let $t_o \leq t \leq t_1$ be a typical subinterval over which data is observed and it is desired to obtain estimates of the wind profile w(t) and initial heading angle $\theta(t_o) = \theta_o$ for purposes of updating the control algorithm on the next subinterval. A general approach to this problem would model w(t) as a stochastic process, perhaps with an underlying Markov process representation, and proceed to derive the partial differential equations from which the conditional means of w(t) and θ_o could be obtained given the data. However, there is little motivation to formulate this full blown version of the estimation problem, at least at this stage of the investigation, due to the rather extensive computational requirements anticipated in solving the partial differential equations. Therefore, in this section the simpler least squares estimation of w(t) and θ_o will be formulated and solved in closed form. Regarding other estimation schemes, a minimum variance

estimate of the wind direction and initial parachute heading will be discussed for the special case in which the magnitude of the wind vector is a known constant.

(a) A Least Squares Estimate

Let the wind components in (1) be modeled by the polynomials of preselected order n:

$$w_1(t) = \sum_{i=0}^{n} \alpha_i t^i$$
$$w_2(t) = \sum_{i=0}^{n} \beta_i t^i \quad . \tag{4}$$

In practical terms n would probably be chosen as either n = 0 (a constant wind of unknown magnitude and direction), or n = 1 (a variable wind with linear time varying components). A least squares estimate of the parameters $(\theta_o, \alpha_o \ldots \alpha_n, \beta_o \ldots \beta_n)$ results upon minimizing the functional

$$J(\theta_o, \alpha, \beta) = \int_{t_o}^{t_1} [\dot{p}_1(t) - a\cos(\theta_o + U(t)) - \sum_{i=0}^{n} \alpha_i t^i]^2 \, dt$$
$$+ \int_{t_o}^{t_1} [\dot{p}_2(t) - a\sin(\theta_o + U(t)) - \sum_{i=0}^{n} \beta_i t^i]^2 dt \tag{5}$$

where U(t) is defined in terms of the bank angle $\phi(t)$ by

$$U(t) = \frac{g}{a} \int_{t_o}^{t} \tan\phi(\tau)\, d\tau \; .$$

A necessary condition for the minimization of (5) is the adherence of the following relations:

$$\frac{\partial J}{\partial \theta_o} = 0, \quad \frac{\partial J}{\partial \alpha_i} = 0, \quad \frac{\partial J}{\partial \beta_i} = 0 \tag{6}$$

$$i = 0, 1 \ldots n \; .$$

Since J is quadratic in the α_i and β_i parameters, the second and third sets of equations in (6) are linear in (α,β) and can be solved uniquely for (α,β) in terms of θ_o and the data. The coefficient matrix for the linear equations in (α,β) is the Gramian for the functions $\{1,t..t^n\}$ on $t_o \leq t \leq t_1$, i.e., the symmetric matrix whose ij-th component (i = 0..n and j = 0..n) is defined by

$$G_{ij} = \int_{t_o}^{t_1} t^{i+j} dt = \frac{t_1^{i+j+1} - t_o^{i+j+1}}{i+j+1} \tag{7}$$

$$0 \leq i,j \leq n \quad .$$

Since $\{1,t..t^n\}$ are linearly independent for any $t_1 > t_o$, the inverse matrix of G exists and can be precomputed and stored for any given $t_o \leq t \leq t_1$ interval. Letting H_{ij} denote the ij-th component of the inverse matrix, G^{-1}, the solutions for α_i and β_i become (details omitted):

$$\alpha_i = \sum_{j=0}^{n} H_{ij}[X_j - a(C_j \cos \theta_o - S_j \sin \theta_o)]$$

$$\beta_i = \sum_{j=0}^{n} H_{ij}[Y_j - a(C_j \sin \theta_o + S_j \cos \theta_o)] \tag{8}$$

$$0 \leq i \leq n$$

where the scalars (C_j,S_j,X_j,Y_j) are given by

$$C_j = \int_{t_o}^{t_1} t^j \cos U(t)dt, \qquad S_j = \int_{t_o}^{t_1} t^j \sin U(t)dt \tag{9}$$

$$X_j = \int_{t_o}^{t_1} t^j \not{p}_1(t)dt, \qquad Y_j = \int_{t_o}^{t_1} t^j \not{p}_2(t)dt \ . \tag{10}$$

Substituting Eq. (8) into the first of the relations in (6) leads to the result

$$\frac{\partial J}{\partial \theta_o} = 0 = A \sin \theta_o - B \cos \theta_o \tag{11}$$

where A and B are defined by

$$A = \int_{t_o}^{t_1} [\dot{p}_1(t) \cos U(t) + \dot{p}_2(t) \sin U(t)]dt$$
$$- \sum_{i=0}^{n} \sum_{j=0}^{n} H_{ij}[X_j C_i + Y_j S_i] \tag{12}$$

and

$$B = \int_{t_o}^{t_1} [\dot{p}_2(t) \cos U(t) - \dot{p}_1(t) \sin U(t)]dt$$
$$+ \sum_{i=0}^{n} \sum_{j=0}^{n} H_{ij}[X_j S_i - Y_j C_i] \tag{13}$$

respectively. Assuming the bank angle $\phi(t)$ is not identically zero on $t_o \leq t \leq t_1$, or equivalently that $U(t)$ is not identically zero, (11) can be solved for θ_o, modulo 2π, taking into account that a minimal value is desired, i.e., taking note of the condition that

$$\frac{\partial^2 J}{\partial \theta_o^2} > 0 \ .$$

This solution is given by

$$\hat{\theta}_o = 2m\pi + \tan^{-1} \frac{B}{A} \tag{14}$$

where m is any integer. Substituting (14) into (8) then yields the final closed-form solution for the least squares estimates of the quantities $(\theta_o, \alpha, \beta)$.

The above solution is contingent on the condition that $\phi(t) \not\equiv 0$ because A and B each vanish if $\phi(t) \equiv 0$ on $t_o \leq t \leq t_1$. In the event that $\phi(t) \equiv 0$ for

all t on $t_o \le t \le t_1$, θ_o cannot be estimated from the given data. In this case a prior value for θ_o should be assumed, based on data collected over a previous subinterval in which $\phi(t) \not\equiv 0$, and $(\hat{\alpha},\hat{\beta})$ can be obtained from

$$\hat{\alpha}_i = \sum_{j=0}^{n} H_{ij}\,(X_j - aG_{jo}\cos\hat{\theta}_o)$$
$$\hat{\beta}_i = \sum_{j=0}^{n} H_{ij}\,(Y_j - aG_{jo}\sin\hat{\theta}_o) \qquad (15)$$

where $\hat{\theta}_o$ is the a priori value assumed for θ_o.

Finally, it should be noted that the integrals involving the total velocity vector of the parachute, $\dot{p}(t)$, in (10), (12) and (13) can be equivalently expressed in terms of $p(t)$ using integration by parts, i.e.,

$$X_j = t_1^{\,j}p_1(t_1) - t_o^{\,j}p_1(t_o) - j\int_{t_o}^{t_1} t^{j-1}p_1(t)dt$$
$$Y_j = t_1^{\,j}p_2(t_1) - t_o^{\,j}p_2(t_o) - j\int_{t_o}^{t_1} t^{j-1}p_2(t)dt \qquad (16)$$

$$\int_{t_o}^{t_1}\dot{p}_i(t)\cos U(t)dt = p_i(t_1)\cos U(t_1) - p_i(t_o) + \frac{g}{a}\int_{t_o}^{t_1} p_i(t)\tan\phi(t)\sin U(t)dt$$
$$\int_{t_o}^{t_1}\dot{p}_i(t)\sin U(t)dt = p_i(t_1)\sin U(t_1) - \frac{g}{a}\int_{t_o}^{t_1} p_i(t)\tan\phi(t)\cos U(t)dt \qquad (17)$$
$$i = 1,2\ .$$

Thus, a knowledge of the data $(p(t), \phi(t))$ on $t_o \le t \le t_1$ is sufficient to obtain the least squares estimate of the wind model (4) and initial heading $\theta(t_o)$.

(b) Statistical Estimates

Although the general estimation problem for a stochastic wind $w(t)$ and random initial heading $\theta(t_o)$ is probably intractable for on-line considerations, there is one special case that leads to a reasonably straightforward solution in computing a minimum variance estimate. This approach involves nonlinear transformations on the data to achieve an underlying linear Markov process in a manner similar to that used by Willsky and Lo [4] for a different but related estimation problem. The stochastic differential equations for this case are assumed as follows:

$$\dot{p}_1(t) = a \cos(\theta(t) + \xi_1(t)) + b \cos(\omega(t) + \xi_2(t))$$
$$\dot{p}_2(t) = a \sin(\theta(t) + \xi_2(t)) + b \sin(\omega(t) + \xi_2(t)) \tag{18}$$

$$d\theta(t) = u(t)dt + d\eta_1(t)$$
$$d\omega(t) = c\omega(t)dt + d\eta_2(t) \; . \tag{19}$$

In the above, the magnitude of the wind vector is assumed to be a known constant parameter "b", $(\xi_1(t), \xi_2(t))$ are independent "white-noise" Gaussian processes, $u(t)$ is a known deterministic forcing function given by

$$u(t) = \frac{g}{a} \tan \phi(t) \; , \tag{20}$$

$(\eta_1(t), \eta_2(t))$ are independent Brownian noise processes, and "c" is a given constant characterizing the transitions for the Markov process $\omega(t)$.

The measurement data is assumed to consist of the total velocity vector of the parachute, $\dot{p}(t)$, as well as the bank angle $\phi(t)$. Equivalently, the data is assumed to consist of the triple of functions $(u(t), z_1(t), z_2(t))$ for $t \geq t_o$ where

$$z_1(t) = \frac{1}{a}\,\hat{p}_1(t) = \cos(\theta + \xi_1) + \rho\cos(\omega + \xi_2)$$

$$z_2(t) = \frac{1}{a}\,\hat{p}_2(t) = \sin(\theta + \xi_1) + \rho\sin(\omega + \xi_2) \tag{21}$$

and $\rho = b/a$ is a known constant. Eliminating the terms involving $(\omega + \xi_2)$ in (21) yields

$$z_1^2 + z_2^2 + 1 - 2||z||\sin\left(\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2}\right) = \rho^2 \tag{22}$$

where $||z|| = [z_1^2 + z_2^2]^{1/2}$. Assuming principal values for the angles, (22) is seen to yield two values for $\theta + \xi_1$ depending on the sign of $\cos\left(\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2}\right)$:

$$\theta + \xi_1 = \begin{cases} -\tan^{-1}\dfrac{z_1}{z_2} + \sin^{-1}\left[\dfrac{z_1^2 + z_2^2 + 1 - \rho^2}{2||z||}\right] & \text{if } \psi > 0 \\[2ex] \pi + \tan^{-1}\dfrac{z_1}{z_2} - \sin^{-1}\left[\dfrac{z_1^2 + z_2^2 + 1 - \rho^2}{2||z||}\right] & \text{if } \psi < 0 \end{cases} \tag{23}$$

where

$$\psi = \operatorname{sgn}\cos\left(\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2}\right). \tag{24}$$

Similarly, the terms involving $(\theta + \xi_1)$ can be eliminated from (21) yielding the scalar equation

$$z_1^2 + z_2^2 + \rho^2 - 2||z||\sin\left(\omega + \xi_2 + \tan^{-1}\frac{z_1}{z_2}\right) = 1\,. \tag{25}$$

Again, two values for $(\omega + \xi_2)$ can be obtained from (25) depending on the sign of $\cos\left(\omega + \xi_2 + \tan^{-1}\frac{z_1}{z_2}\right)$, (assuming principal values for all angles):

$$\omega + \xi_2 = \begin{cases} -\tan^{-1}\dfrac{z_1}{z_2} + \sin^{-1}\left[\dfrac{z_1^2 + z_2^2 + \rho^2 - 1}{2||z||}\right] & \text{if } \phi > 0 \\ \pi + \tan^{-1}\dfrac{z_1}{z_2} - \sin^{-1}\left[\dfrac{z_1^2 + z_2^2 + \rho^2 - 1}{2||z||}\right] & \text{if } \phi < 0 \end{cases} \tag{26}$$

where

$$\phi = \text{sgn} \cos (\omega + \xi_2 + \tan^{-1}\frac{z_1}{z_2}) . \tag{27}$$

The ambiguity in the expressions (23) and (26) cannot be resolved in any simple way. However, considering the time derivative of $\sin (\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2})$:

$$\frac{d}{dt}\sin (\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2}) = \cos (\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2})\frac{d}{dt}(\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2})$$

$$\approx u(t) \cos (\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2}) .$$

The latter approximation holds if the angular rate term $\frac{d}{dt}(\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2})$ is dominated by $\dot{\theta}(t) = u(t)$. Then the function ψ in (24) becomes

$$\psi = \{\text{sgn } u(t)\} \text{ sgn } \frac{d}{dt}\sin(\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2}) . \tag{28}$$

But sgn $\{\frac{d}{dt}\sin (\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2})\}$ can be expressed in terms of $z(t)$ and $\dot{z}(t)$ by differentiating (22) and assuming $||z|| > 0$:

$$\text{sgn } \{\frac{d}{dt}\sin (\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2})\} = \text{sgn } \{(z_1\dot{z}_1 + z_2\dot{z}_2)(z_1^2 + z_2^2 - 1 + \rho^2)\}. \tag{29}$$

This implies that the value of ψ in (28) can be resolved if the sign of the quantity in brackets on the right side of (29) can be determined from the measurements. However, it should be reiterated that this result depends on the assumption that $\dot{\theta}(t) = u(t)$ dominates the angular rate $\frac{d}{dt}(\theta + \xi_1 + \tan^{-1}\frac{z_1}{z_2})$.

The value of ϕ in (27) can be related to the value of ψ from the following trigonometric considerations. Let $\lambda = \tan^{-1} \frac{z_1}{z_2}$ so that

$$\cos \lambda = \frac{z_2}{||z||}, \quad \sin \lambda = \frac{z_1}{||z||} .$$

Then the following identity follows from (21):

$$\begin{aligned} z_1 \cos \lambda - z_2 \sin \lambda &= \cos \lambda \cos(\theta + \xi_1) + \rho \cos \lambda \cos(\omega + \xi_2) \\ &\quad - \sin \lambda \sin(\theta + \xi_1) - \rho \sin \lambda \sin(\omega + \xi_2) \\ &= \cos[\lambda + \theta + \xi_1] + \rho \cos[\lambda + \omega + \xi_2] \\ &= 0 . \end{aligned} \tag{30}$$

But ρ is positive so that $\phi = -\psi$, which resolves the ambiguity in (26) once ψ is determined.

Given the nonlinear transformations on the data so that the right hand sides of Eqs. (23) and (26) are known at each instant of time t, the second pair of equations in (19) can now be regarded as a vector Markov process with <u>linear measurements</u> as summarized by the following matrix equations:

$$d \begin{bmatrix} \theta \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & c \end{bmatrix} \begin{bmatrix} \theta \\ \omega \end{bmatrix} dt + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \, dt + d \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \tag{31}$$

$$\begin{bmatrix} \tilde{z}_1 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \omega \end{bmatrix} + \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \tag{32}$$

where $\tilde{z}_1$ and $\tilde{z}_2$ denote the right hand sides of (23) and (26), respectively.

Equations (31) and (32) are now in the standard form for application of the Kalman-Bucy filter [5] in obtaining a minimum variance estimate of the pair $(\theta(t), \omega(t))$ conditioned on the data $(\tilde{z}_1(t), \tilde{z}_2(t))$. The filter for this estimate is given by

$$d\hat{x} = A\hat{x}\,dt + Bu\,dt + K(t)[\tilde{z} - \hat{x}]dt, \quad \hat{x}(o) = E(x_o) \tag{33}$$

where $\hat{x} = (\hat{\theta}, \hat{\omega})'$; A and B are the coefficient matrices in (31), and the gain matrix K(t) is computed off-line according to

$$K(t) = P(t)R_2^{-1}$$
$$\frac{dP}{dt} = AP + PA' + R_1 - PR_2^{-1}P \ , \quad P(o) = E(x_o x_o') \tag{34}$$

where

$$R_1 dt = E(\eta\eta')$$

and

$$R_2 = E(\xi\xi')$$

are presumed to be given covariance matrices with R_2 positive definite. Equation (33) is the real-time realization of this optimal filter given that the gain matrix K(t) has been pre-computed off-line by the integration of the Riccati differential equation in (34).

III. CLOSED LOOP CONTROL ALGORITHM

Given estimates of the wind vector, $(w_1(t), w_2(t))$, over $t_o \leq t \leq T$, and the initial heading of the parachute relative to wind, $\theta(t_o)$, as determined by the least squares formulae of Section II-a involving the data observed over the previous subinterval, the following transformations to normalized coordinates simplify the kinematic equations for control considerations:

$$x_i(t) = \frac{1}{(T-t_o)a}[p_i(t) + \int_t^T w_i(\xi)d\xi], \quad i = 1,2$$
$$x_3(t) = \theta(t) \ . \tag{35}$$

Rewriting Eq. (1) in terms of (x_1, x_2, x_3) and introducing the normalized time τ,

$$\tau = \frac{t-t_o}{T-t_o} \ , \tag{36}$$

and the normalized control variable u,

$$u = \frac{(T-t_o)g}{a} \tan \phi , \tag{37}$$

the kinematic equations become

$$\begin{aligned} \dot{x}_1(\tau) &= \cos x_3(\tau) \\ \dot{x}_2(\tau) &= \sin x_3(\tau) \qquad 0 \leq \tau \leq 1 \\ \dot{x}_3(\tau) &= u(\tau) . \end{aligned} \tag{38}$$

The desired terminal state in these coordinates is given by:

$$x_1(1) = x_2(1) = 0, \quad x_3(1) = \underline{/w(T)} + \pi \tag{39}$$

where $\underline{/w(T)}$ denotes the estimated wind direction at the terminal time T.

The optimal control problem of minimizing the control energy, $\int_o^1 |u(\tau)|^2 d\tau$, while driving the system (38) from the initial state

$$\begin{aligned} x_i(o) &= \frac{1}{(T-t_o)a} [p_i(t_o) + \int_{t_o}^{T} w_i(\xi)d\xi], \quad i = 1,2 \\ x_3(o) &= \theta(t_o) \end{aligned} \tag{40}$$

to the terminal state (39) has been investigated in [2] and [3]. Assuming the initial coordinates $(x_1(o), x_2(o))$ lie within the unit circle, this is a well posed problem with moderately demanding computational requirements in obtaining a solution. The Differential Dynamic Programming algorithm for computing the optimal control, as discussed in [2], requires a large amount of computer storage, but tends to converge in a small number iterations. The parameter search algorithm, discussed in [3] and further investigated via the application of the Davison-Wong technique [6], requires far less memory, but requires many more iterations to converge.

Although each of the optimal control techniques may be feasible if sufficient computer hardware is available, the far simpler bang-off-bang algorithm

described in Section VI of [3] was utilized for the control algorithm in closing the loop using the step by step estimation-control sequence described in the Introduction. However, provision in this algorithm must be made for the possibility that the initial conditions in (40) may lie outside the unit circle at the start of any particular sub-interval, thereby necessitating an alternative control strategy (not discussed in [2] or [3]) for this situation.

(a) Control Strategy for Initial Conditions Outside the Unit Circle

The following control strategy was adopted for the case in which $(x_1(o), x_2(o))$ lie outside the unit circle. Let $u(\tau)$ be constrained to be either one of the two forms:

$$u_1(\tau) = \begin{cases} \frac{1}{\gamma} & \text{for } 0 \leq \tau \leq t_1 \\ 0 & \text{for } t_1 < \tau \leq 1 \end{cases} \tag{41}$$

or

$$u_2(\tau) = \begin{cases} 0 & \text{for } 0 \leq \tau \leq t_1 \\ \frac{1}{\gamma} & \text{for } t_1 < \tau \leq 1 \end{cases} \tag{42}$$

where the normalized turning radius, γ, and the switching time t_1 are to be determined by minimizing the function

$$J(t_1) = [x_1^2(1) + x_2^2(1)] \tag{43}$$

subject to the end-point constraint

$$x_3(1) = \underline{/w(T)} + \pi \,. \tag{44}$$

Using the control u_1 in (41), the equations of motion (38) can be integrated yielding an explicit expression for $J(t_1)$. The terminal constraint (44) implies the following relation between γ and t_1:

$$\gamma = \frac{t_1}{\underline{/w(T)} + \pi - x_3(o)} \,. \tag{45}$$

Using this constraint and the necessary condition for a minimum, $\frac{dJ}{dt_1} = 0$, the following values for t_1^* and $J^* = J(t_1^*)$ are obtained:

$$t_1^* = \frac{1}{d}\Big\{1 + x_1(o)\cos\nu + x_2(o)\sin\nu + \frac{1}{x_3(o)-\nu}\,[x_1(o)\sin\nu - x_2(o)\cos\nu - x_1(o)\sin x_3(o) + x_2(o)\cos x_3(o) + \sin(\nu - x_3(o))]\Big\} \tag{46}$$

$$J_1^* = \frac{1}{d}\Big\{x_2(o)\cos\nu - x_1(o)\sin\nu + \frac{1}{\nu - x_3(o)}\,[x_1(o)\cos x_3(o) + x_2(o)\sin x_3(o) + \cos(\nu - x_3(o)) - 1 - x_1(o)\cos\nu - x_2(o)\sin\nu]\Big\}^2 \tag{47}$$

where d and ν are defined by

$$\nu = \underline{/w(T)} + \pi \tag{48}$$

$$d = \frac{[\nu - x_3(o) - \sin(\nu - x_3(o))]^2 + 4\sin^4\left(\frac{\nu - x_3(o)}{2}\right)}{[\nu - x_3(o)]^2} \,. \tag{49}$$

With the above value for t_1^*, it can be shown that $\frac{d^2J}{dt_1^2} > 0$ so that t_1^* is a minimal point. This implies that u_1 in (41) will be the proper control to apply (within the present context) provided, in addition, that $0 \leq t_1^* \leq 1$ and $\nu \neq x_3(o)$.

In a similar manner, the differential equations can be integrated using the control u_2 in (42) resulting in an explicit relation for $J(t_1)$. Again, the terminal constraint (44) implies the following constraint between the radius of turn γ and t_1 (cf. (45)):

$$\gamma = \frac{1 - t_1}{\underline{/w(T)} + \pi - x_3(o)} \,. \tag{50}$$

The minimizing value of t_1 and corresponding minimal value of J in this case is found to be

$$t_1^* = \frac{1}{d}\left\{\frac{2 - 2\cos(\nu - x_3(o))}{[\nu - x_3(o)]^2} + \frac{1}{\nu - x_3(o)}[x_1(o)\sin\nu - x_2(o)\cos\nu\right.$$

$$+ x_2(o)\cos x_3(o) - x_1(o)\sin x_3(o) - \sin(\nu - x_3(o))]$$

$$\left. - x_1(o)\cos x_3(o) - x_2(o)\sin x_3(o)\right\} \tag{51}$$

and

$$J_2^* = \frac{1}{d}\left\{x_1(o)\sin x_3(o) - x_2(o)\cos x_3(o) + \frac{1}{\nu - x_3(o)}[x_1(o)\cos\nu\right.$$

$$\left. + x_2(o)\sin\nu - x_1(o)\cos x_3(o) - x_2(o)\sin x_3(o) - 1 + \cos(\nu - x_3(o))]\right\}^2 \tag{52}$$

As in the previous case, u_2 is feasible only if t_1^* in (51) satisfies $0 \leq t_1^* \leq 1$. In practice, both cases must be considered for any particular set of initial values $(x_1(o), x_2(o))$ lying outside the unit circle with the choice, u_1 or u_2, based on feasibility. It could be that neither case is feasible for certain initial data in which case the value of J can be computed for full on, or full off, control during $0 \leq \tau \leq 1$, and that control selected which achieves the smaller value for J, consistent with the end point heading constraint (44). These details of the control strategy have been programmed into the Fortran listing supplied in the Appendix.

(b) Simulation Results of the Closed Loop Controller

Simulation studies were carried out for the system (1) using a variety of initial conditions $(p_1(o), p_2(o), \theta(o))$ and wind profiles $(w_1(t), w_2(t))$ over the total time interval $0 \leq t \leq 307.5$ sec. The speed of the parachute relative to wind was fixed at a = 30 ft/sec. Five subintervals were used for the step-by-step estimation-control sequence with the lengths of these subintervals defined by:

$$t_1 = 7.5, \quad t_3 = 157.5$$
$$t_2 = 82.5, \quad t_4 = 232.5$$
$$T = 307.5 \ .$$

A small control effort of magnitude 0.01 was exerted over the first subinterval in order to avoid the degeneracy discussed at the end of Section II-a in estimating the parachute heading θ_o.

All integrations were performed using a fourth order Runge-Kutta subroutine from the IBM Scientific Subroutine Package. A complete Fortran listing of the computer program is given in the Appendix. The differential equations for the parachute, the generation of the wind vector, as well as all the relevant integrals needed for the least squares estimation are integrated in the subroutine labeled CPLANT. A linear time varying wind model was used in the wind estimation subroutine (Eq. (4) with n = 1):

$$w_1(t) = \alpha_o + \alpha_1 t$$
$$w_2(t) = \beta_o + \beta_1 t \ .$$

The actual winds used in the study are given in Table 1. The analytical expressions for both polar and rectangular coordinates of the wind vector are indicated. A step-type disturbance was introduced for some of the runs as indicated by the Δw_i columns in Table 1. These disturbances (where indicated) were imposed at the end of each subinterval according to the rule:

$$w_i(\text{new}) = w_i(\text{old}) + \Delta w_i, \quad i = 1,2 \ .$$

The parachute trajectories under closed loop control are shown in Figs. 1-11 with corresponding plots for the wind profile and the parachute bank angle. Two different trajectories are shown on each Figure corresponding to the two different sets of initial conditions indicated. The terminal error, $||p(T)||$, is the Euclidean distance in feet, while $\Delta\theta(T)$ denotes the error in the desired parachute heading at the terminal time. These trajectories and data

indicate that good terminal accuracy can be obtained for smooth variable winds, with some deterioriation in accuracy for abruptly shifting winds. The bank angles for the most part are quite reasonable, although there were brief moments where bank angles in excess of 30° were called for by the control algorithm. There was no attempt to determine the best sizing of subintervals, nor to experiment with variations in the estimation scheme. Such experimentation is necessary if a practical implementation of this approach is undertaken.

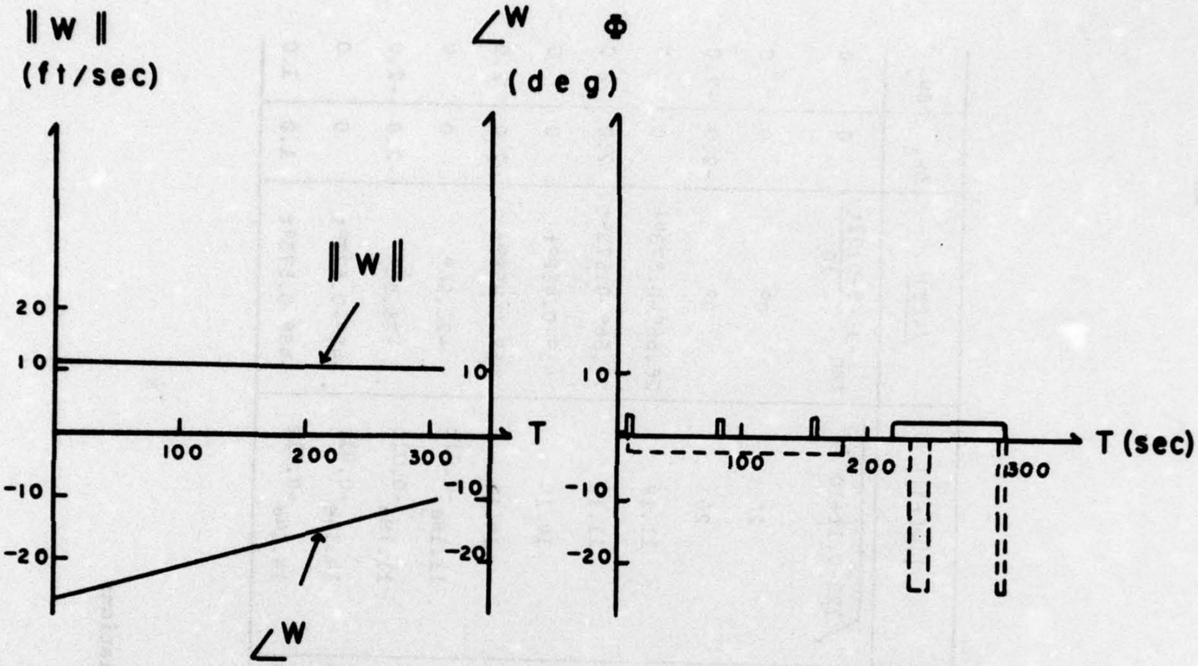
IV. CONCLUSIONS

Separating the wind and initial heading estimation problems from the control problem to obtain a step-by-step estimation and control sequence may be a feasible approach to the gliding parachute control problem in a nonuniform wind. It will be difficult to make a more definitive statement until additional simulations and experimentations are carried out. Even within the scope of the relatively simple least squares estimation scheme used in this study, additional experimentation is needed to determine the number and sizing of subintervals $t_i \leq t \leq t_{i+1}$, whether or not to combine wind estimates over adjacent subintervals by averaging the estimates over several subintervals, and what form of wind model to use in the estimation scheme. The control aspect of the problem is fairly straightforward from a computational viewpoint, but actuator dynamics have been completely neglected as indicated by the instantaneous step changes allowed in the parachute bank angle. More sophisticated estimation and control algorithms might offer better performance, but at the expense of more complex computations.

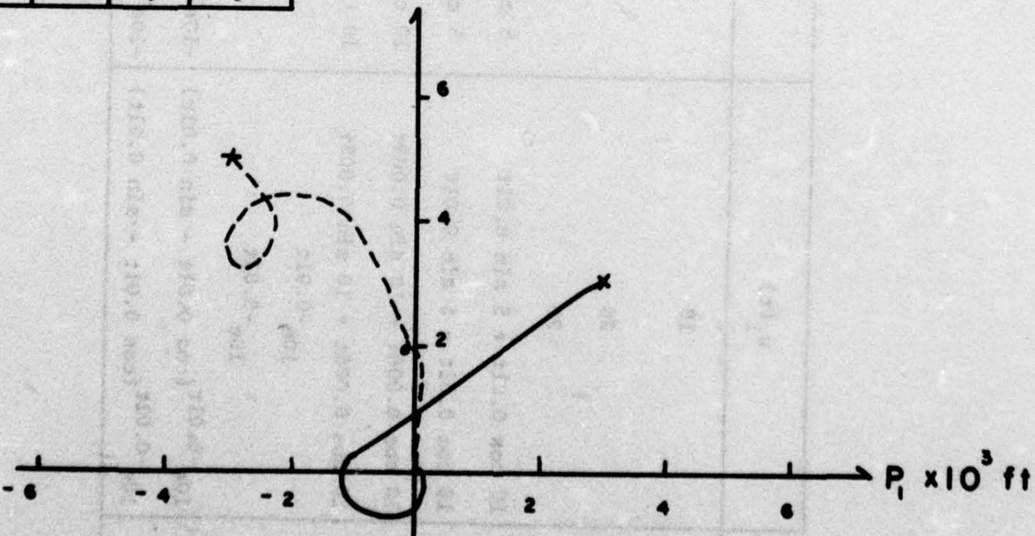
Wind No.	$w_1(t)$	$w_2(t)$	$\|\|w(t)\|\|$	$\angle w(t)$	Δw_1	Δw_2
1	10	$-5 + 0.01t$	$\sqrt{125-0.1t+10^{-4}t^2}$	$\tan^{-1} \frac{-5+0.01t}{10}$	0	0
2	20	0	20	$0°$	0	0
3	20	0	20	$0°$	-2.0	-2.0
4	$10 \cos 0.01t + 5 \sin 0.01t$	$5 \cos 0.01t - 10 \sin 0.01t$	11.18	$26.56° - 0.573°t$	0	0
5	$10 \cos 0.01t + 5 \sin 0.01t$	$5 \cos 0.01t - 10 \sin 0.01t$	11.18	$26.56° - 0.573°t$	2.0	2.0
6	$10 \cos 0.008t + 10 \sin 0.008t$	$10 \cos 0.008t - 10 \sin 0.008t$	14.14	$45° - 0.458°t$	0	0
7	$10 \cos 0.008t + 10 \sin 0.008t$	$10 \cos 0.008t - 10 \sin 0.008t$	14.14	$45° - 0.458°t$	2.0	2.0
8	$10e^{-0.01t}$	$-5e^{-0.01t}$	$11.18e^{-0.01t}$	$-26.56°$	0	0
9	$10e^{-0.01t}$	$-5e^{-0.01t}$	$11.18e^{-0.01t}$	$-26.56°$	-2.0	-2.0
10	$10e^{-0.01t}(\cos 0.01t - \sin 0.01t)$	$-10e^{-0.01t}(\cos 0.01t + \sin 0.01t)$	$14.14e^{-0.01t}$	$-45° - 0.573°t$	0	0
11	$10e^{-0.01t}(\cos 0.01t - \sin 0.01t)$	$-10e^{-0.01t}(\cos 0.01t + \sin 0.01t)$	$14.14e^{-0.01t}$	$-45° - 0.573°t$	1.0	1.0

TABLE I

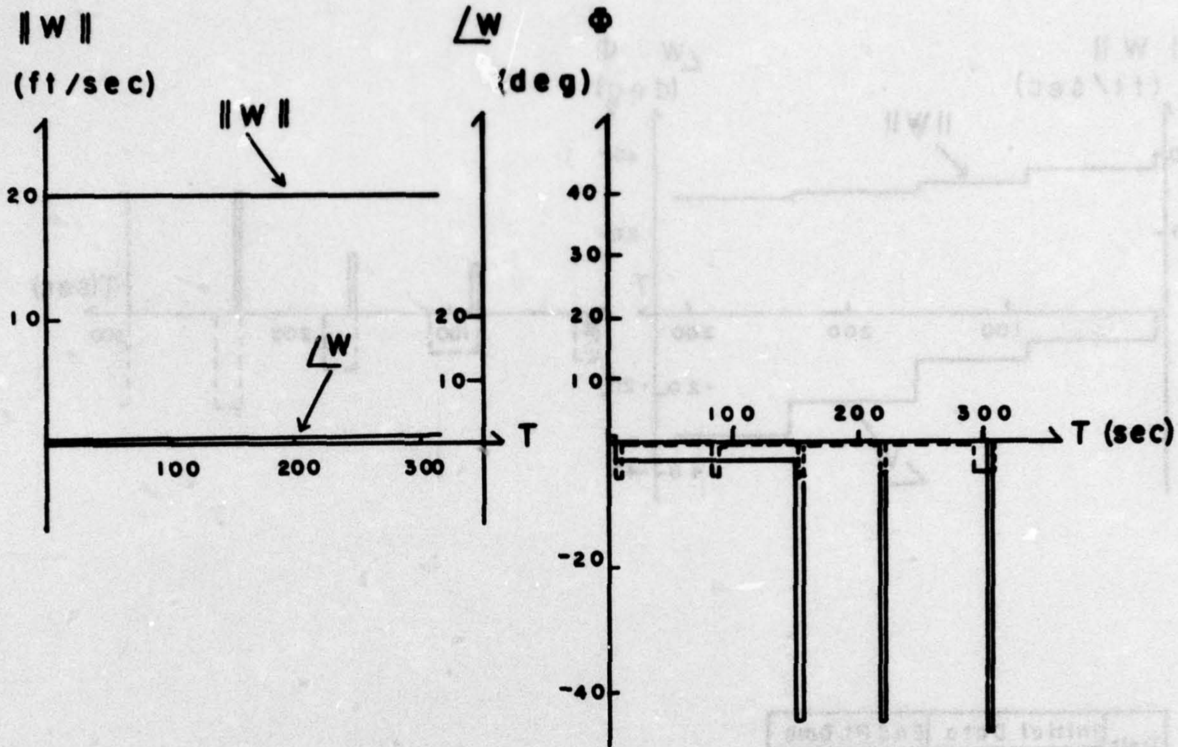
Actual Wind Profiles for the Simulations



Trajs	Initial Data		EndPt. Data	
	‖P(0)‖	θ(0)	‖ρ(T)‖	Δθ(T)
——	4243	180°	0	0°
- - -	5830	-45°	0	0°



<u>Fig. 1</u> Simulation Data for Closed-Loop Control, Wind No. 1



Trajs	Initial Data		End Pt. Data	
	‖P(0)‖	θ(0)	‖P(T)‖	Δθ(T)
———	7071	-45°	4	0°
- - -	13796	-45°	0	0°

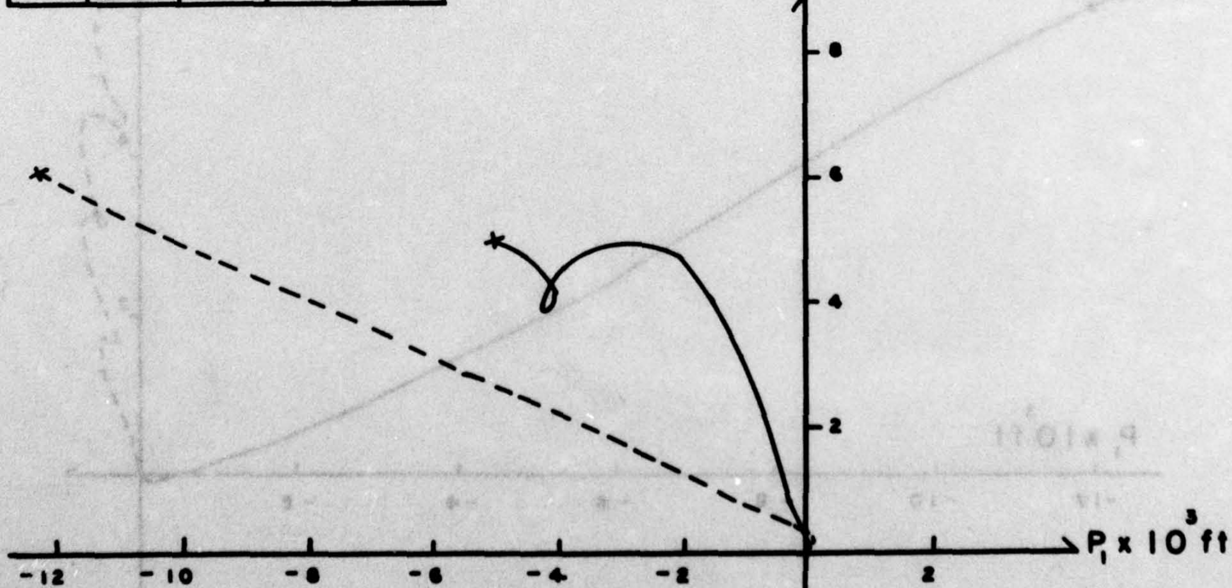
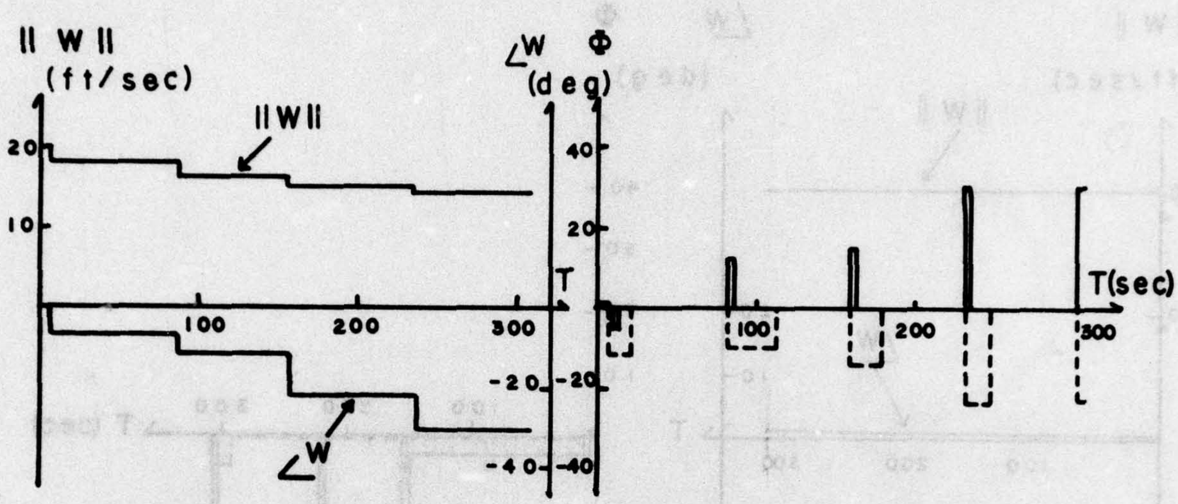


Fig. 2 Simulation Data for Closed-Loop Control, Wind No. 2



Trajs	Initial Data $\|P(0)\|$	$\theta(0)$	End Pt. Data $\|P(T)\|$	$\Delta\theta(T)$
——	13796	-45°	210	-8°
---	6100	-0°	211	-10°

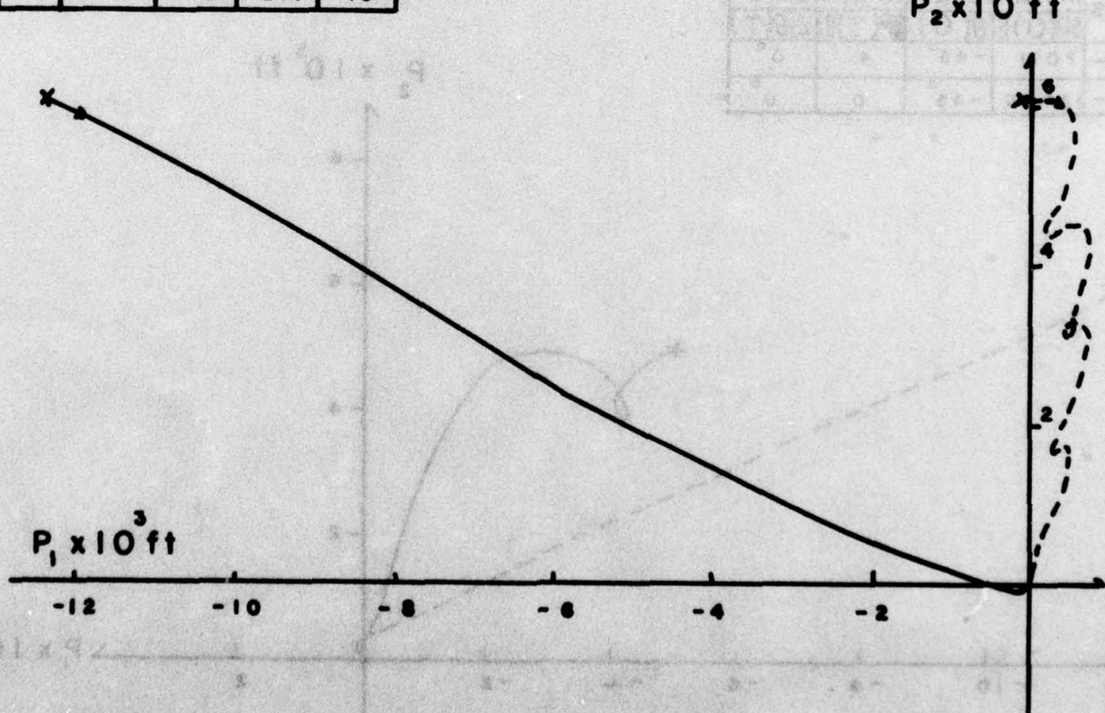
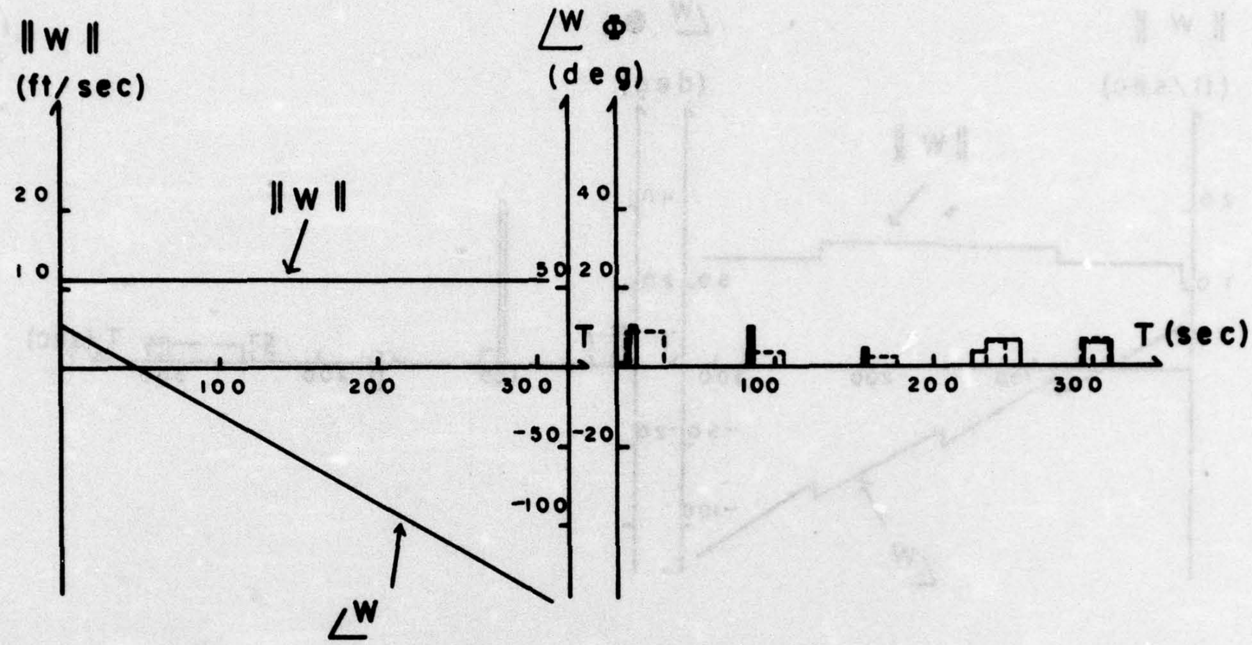


Fig. 3 Simulation Data for Closed-Loop Control, Wind No. 3



Trajs	Initial Data		End Pt. Data	
	‖P(O)‖	θ(O)	‖P(T)‖	Δθ(T)
———	4460	90°	236	16°
– – –	6100	-45°	252	20°

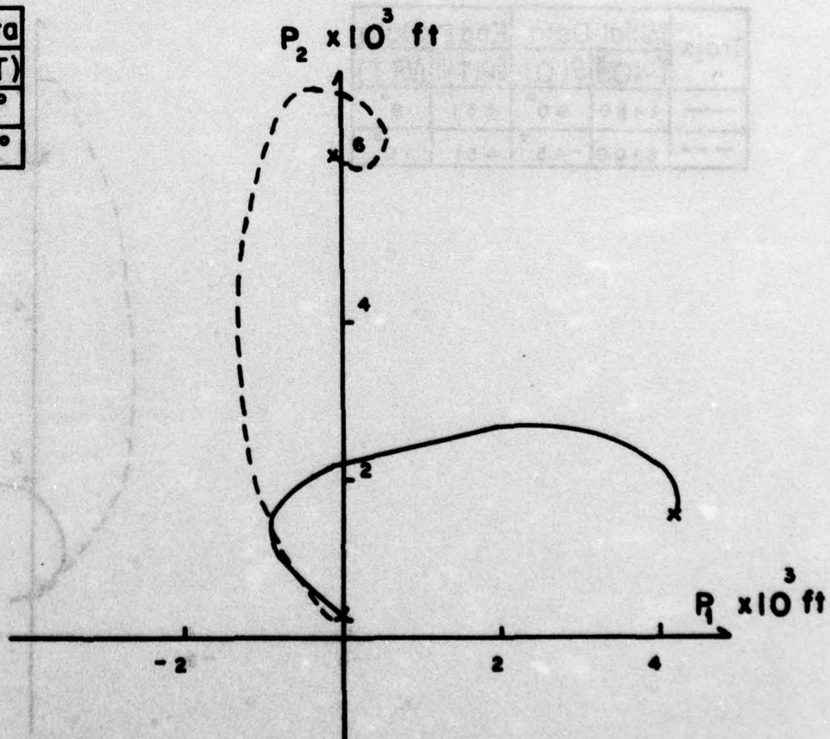
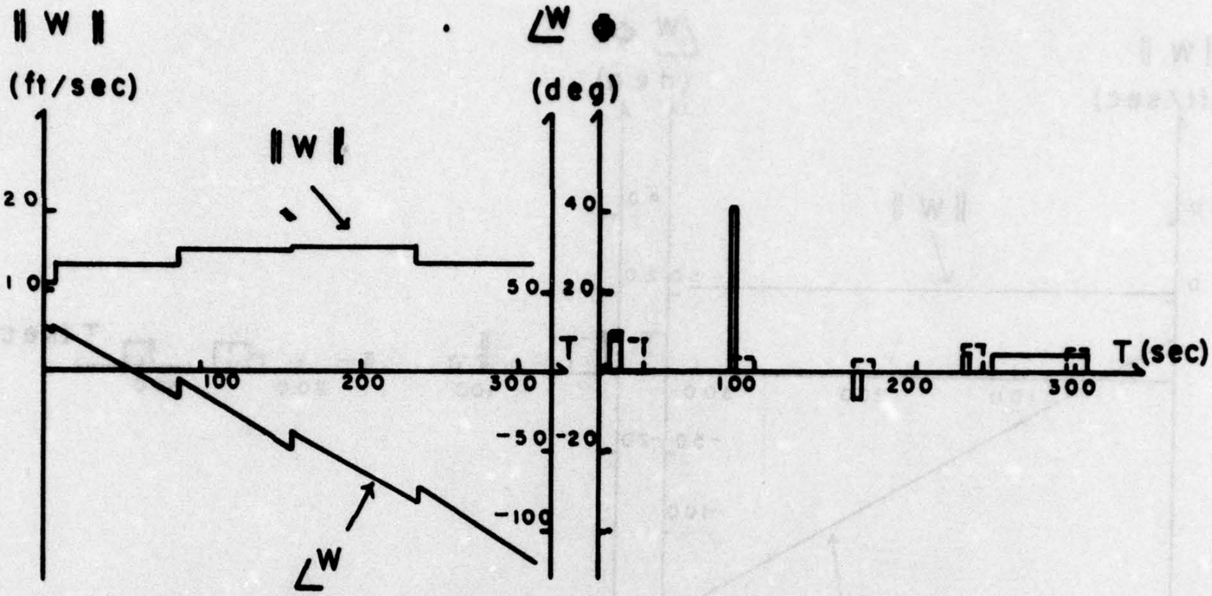


Fig. 4 Simulation Data for Closed-Loop Control, Wind No. 4



Trajs	Initial Data		End Pt. Data	
	‖P(O)‖	θ(O)	‖P(T)‖	Δθ(T)
——	4460	90°	551	9°
- - -	6100	-45°	451	10°

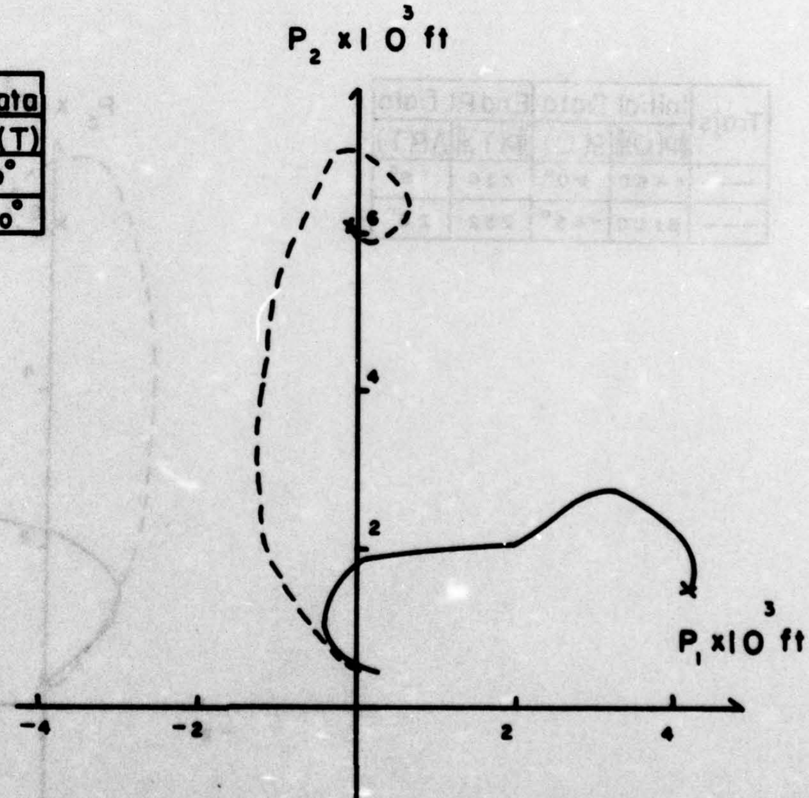
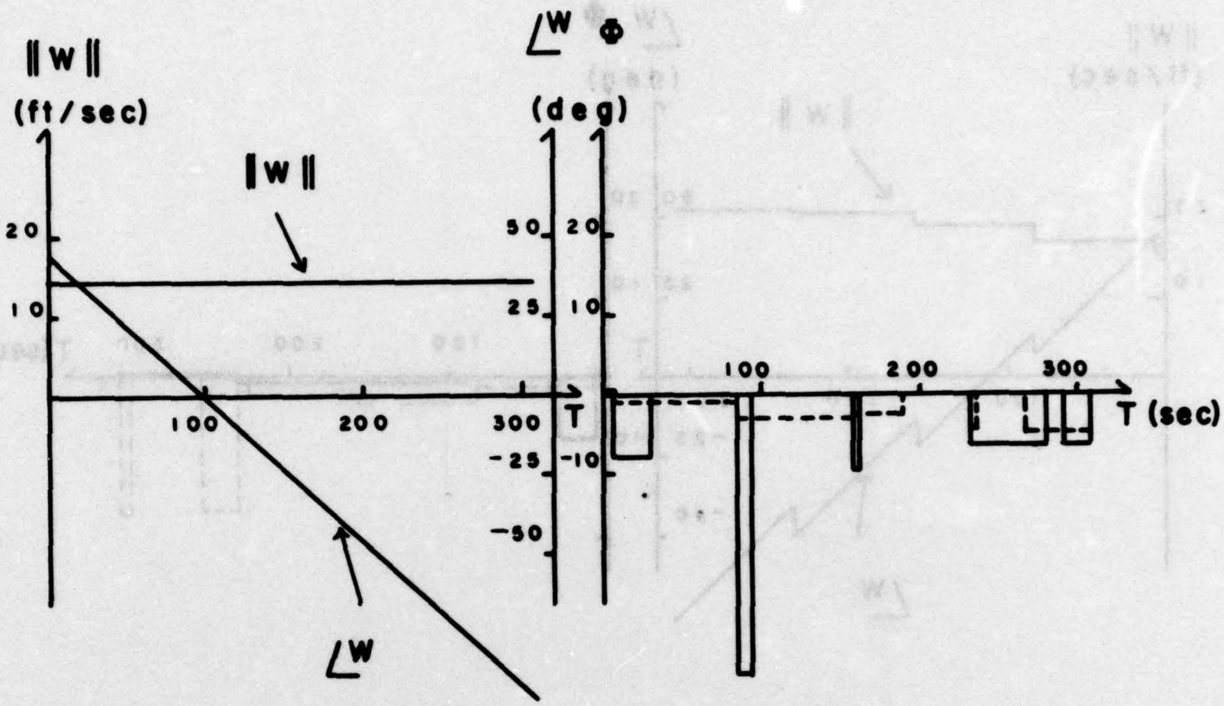


Fig. 5 Simulation Data for Closed-Loop Control, Wind No. 5



Trajs	Initial Data		End Pt Data	
	‖P(O)‖	θ(O)	‖P(T)‖	Δθ(T)
——	3535	45°	244	-350°
----	7071	0°	161	-356°

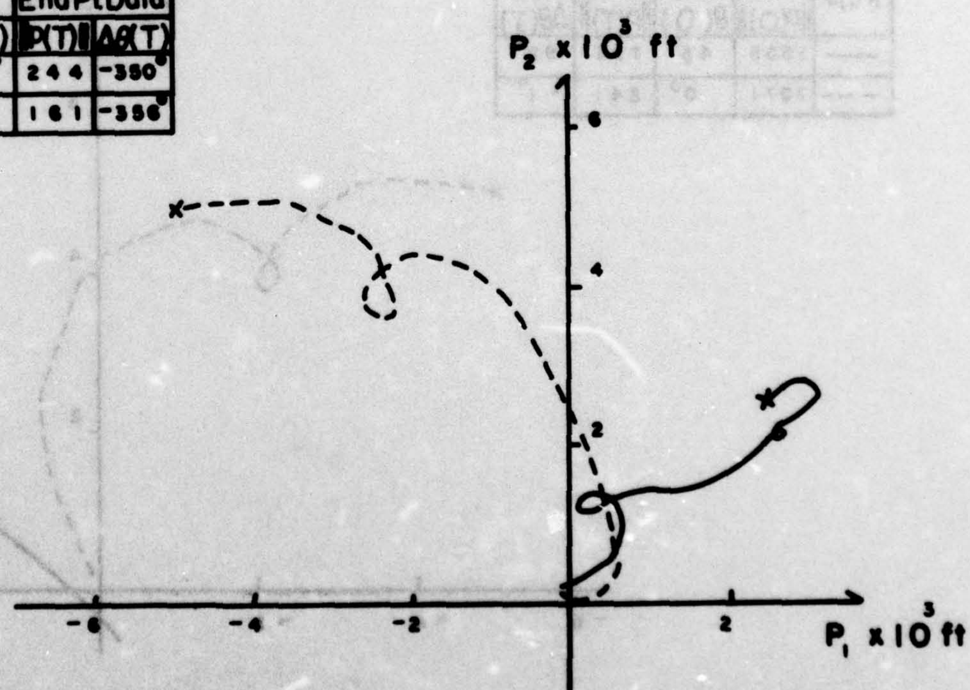
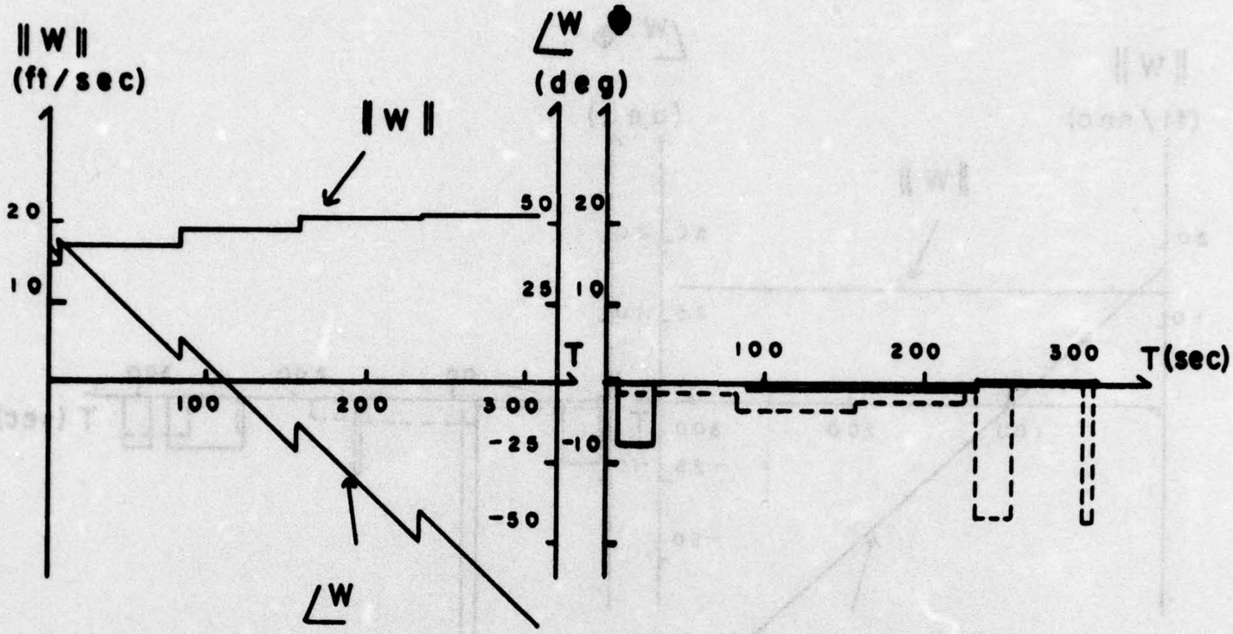


Fig. 6 Simulation Data for Closed-Loop Control, Wind No. 6



Trajs	Initial Data		End Pt. Data	
	$\|P(0)\|$	$\theta(0)$	$\|P(T)\|$	$\Delta\theta(T)$
——	3535	45°	755	92°
- - -	7071	0°	241	-1°

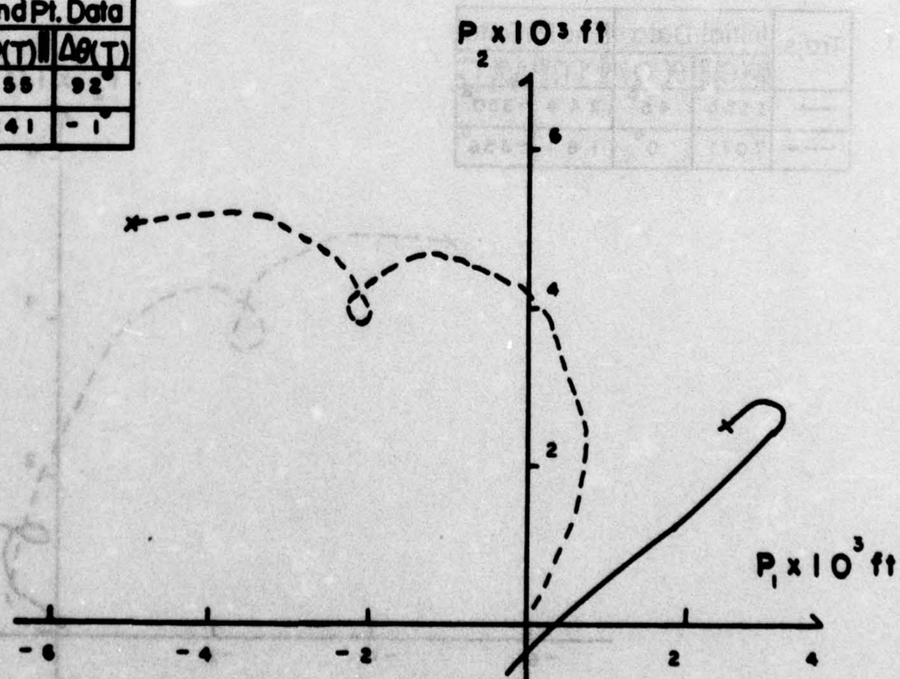
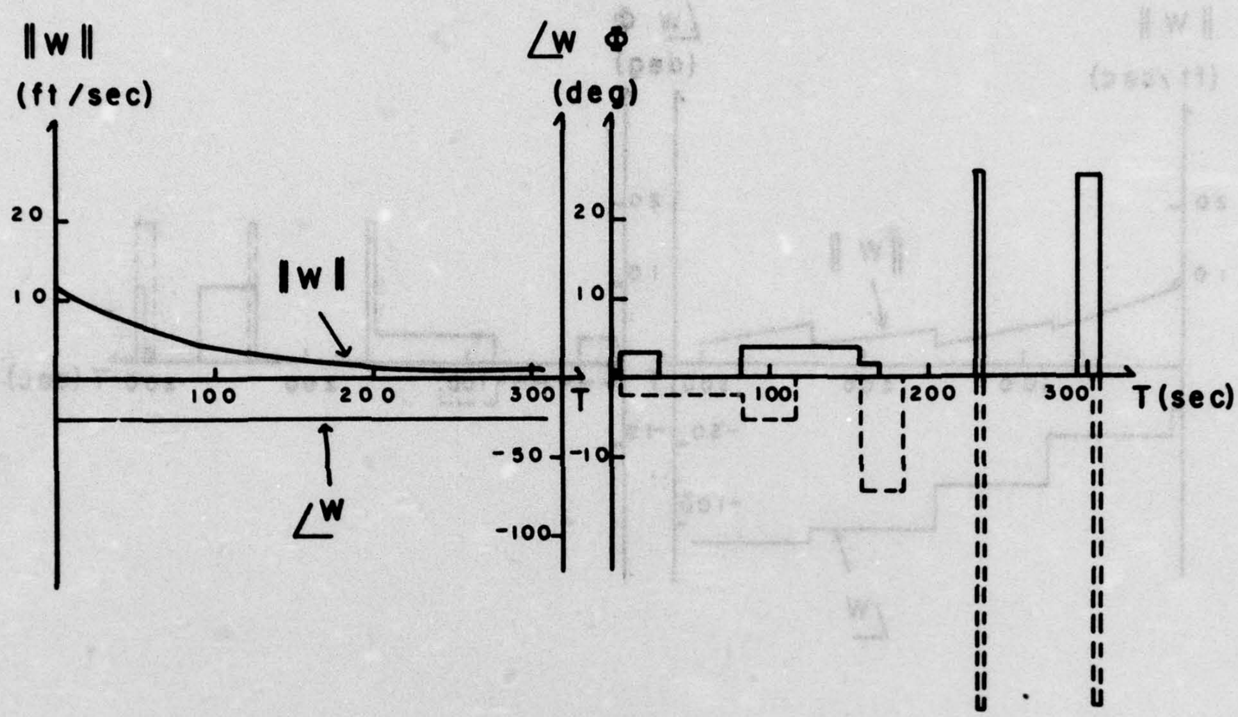


Fig. 7 Simulation Data for Closed-Loop Control, Wind No. 7



Trajs	Initial Data		End Pt. Data	
	$\lVert p(0) \rVert$	$\theta(0)$	$\lVert p(T) \rVert$	$\Delta\theta(T)$
———	6082	180°	24	176°
- - -	6082	0°	25	167°

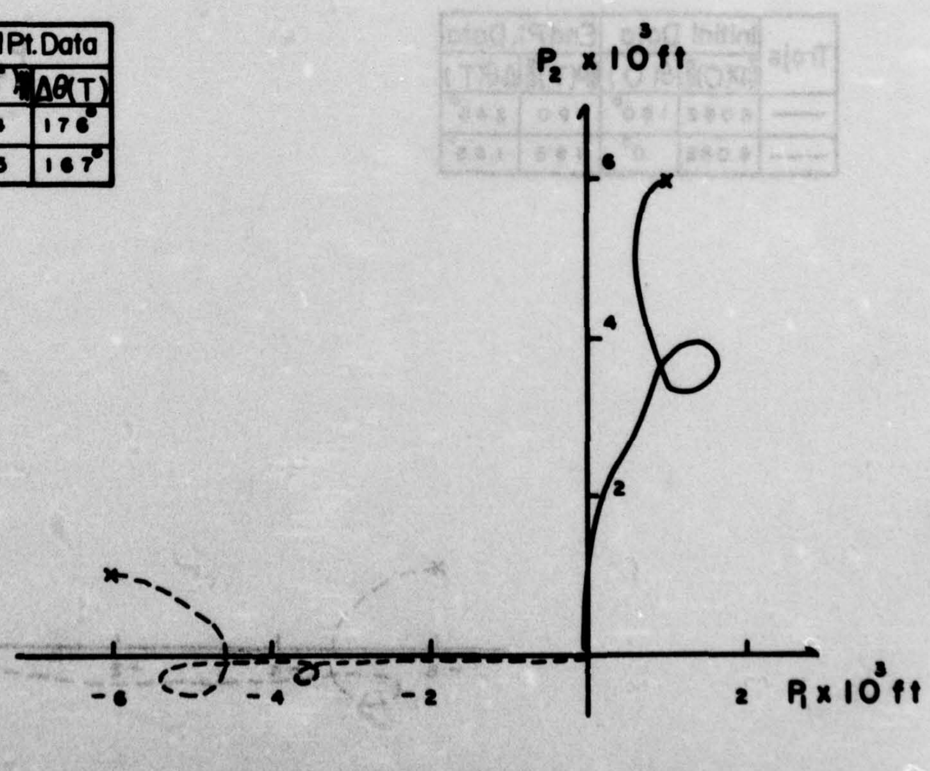
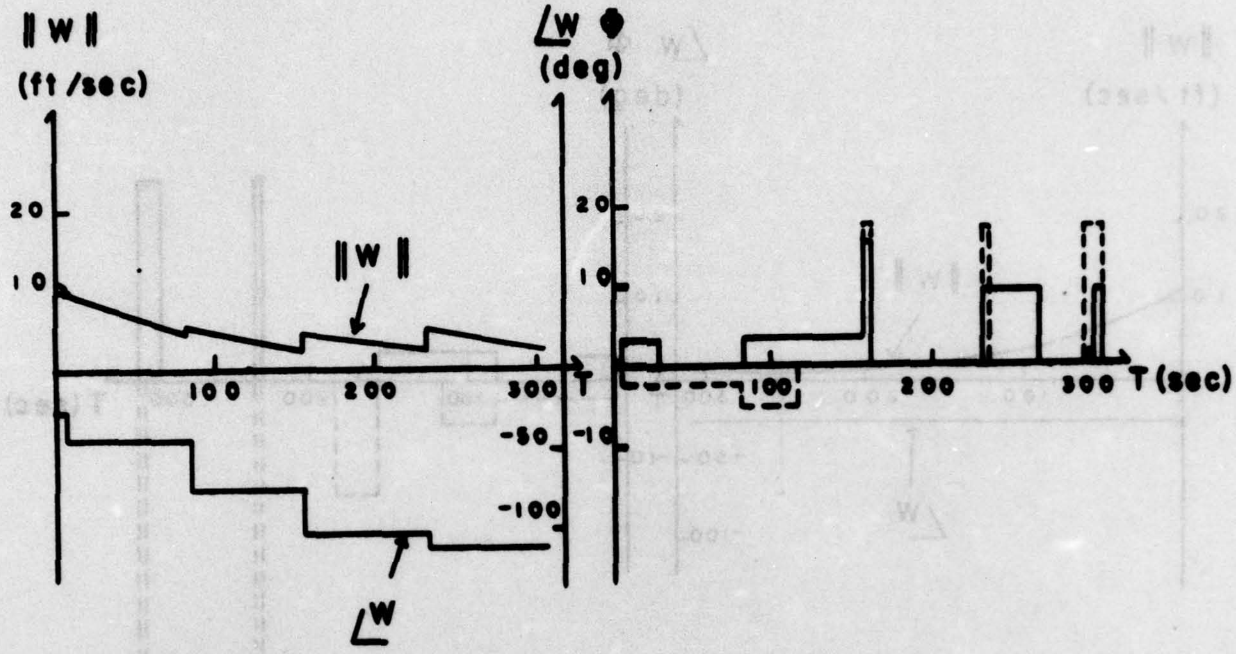


Fig. 8 Simulation Data for Closed-Loop Control, Wind No. 8



Trajs	Initial Data		End Pt. Data	
	$\lVert P(0) \rVert$	$\theta(0)$	$\lVert P(T) \rVert$	$\Delta\theta(T)$
——	6062	180°	190	245°
– – –	6062	0°	193	185°

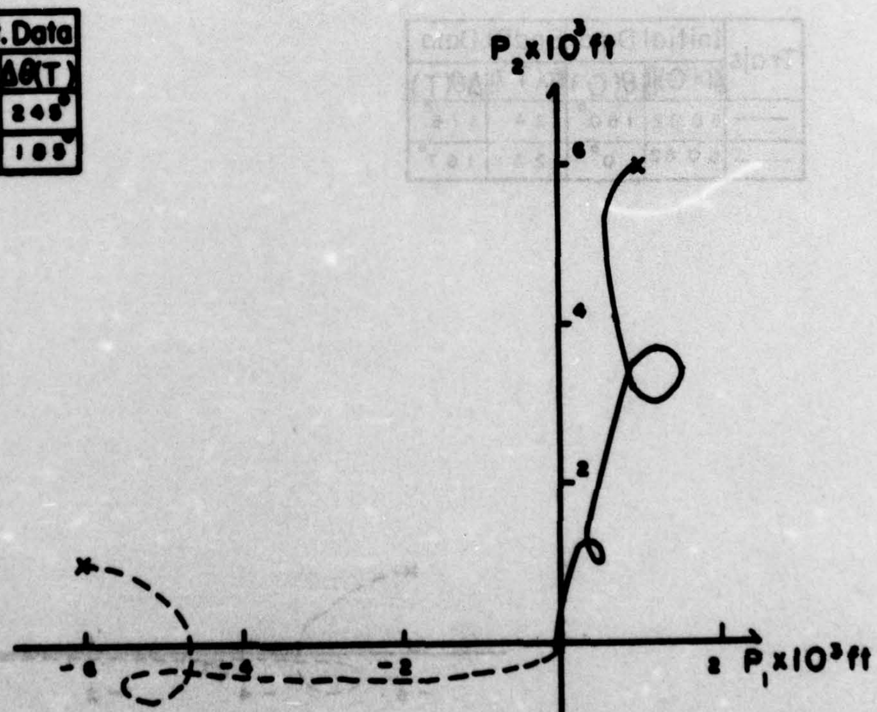
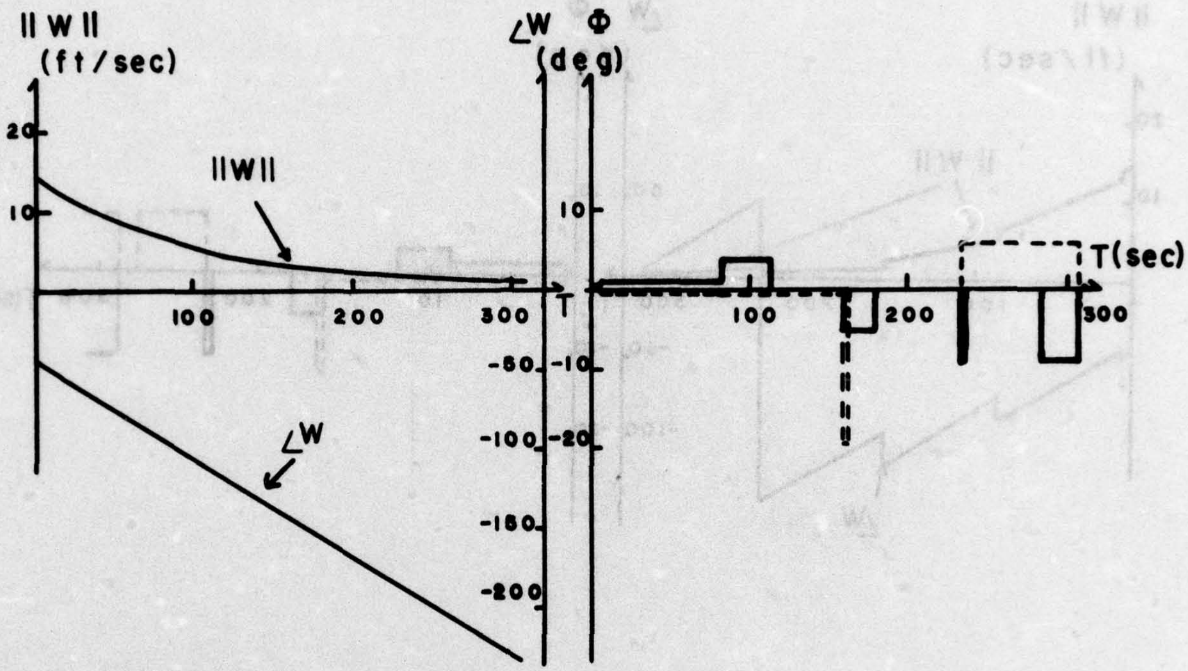


Fig. 9 Simulation Data for Closed-Loop Control, Wind No. 9



Trajs	Initial Data		End Pt.Data	
	$\|P(0)\|$	$\theta(0)$	$\|P(T)\|$	$\Delta\theta(T)$
——	4460	90°	57	-26°
- - -	6100	-45°	58	330°

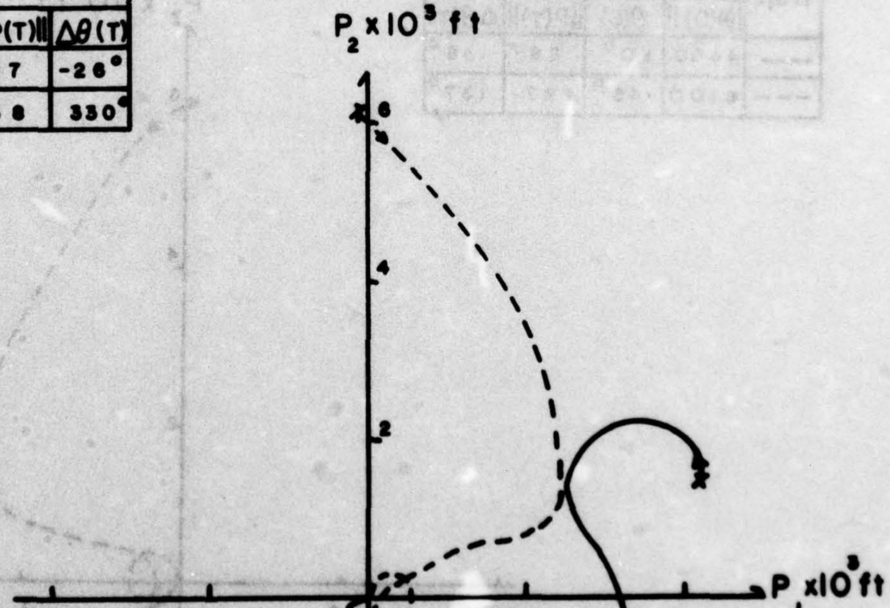
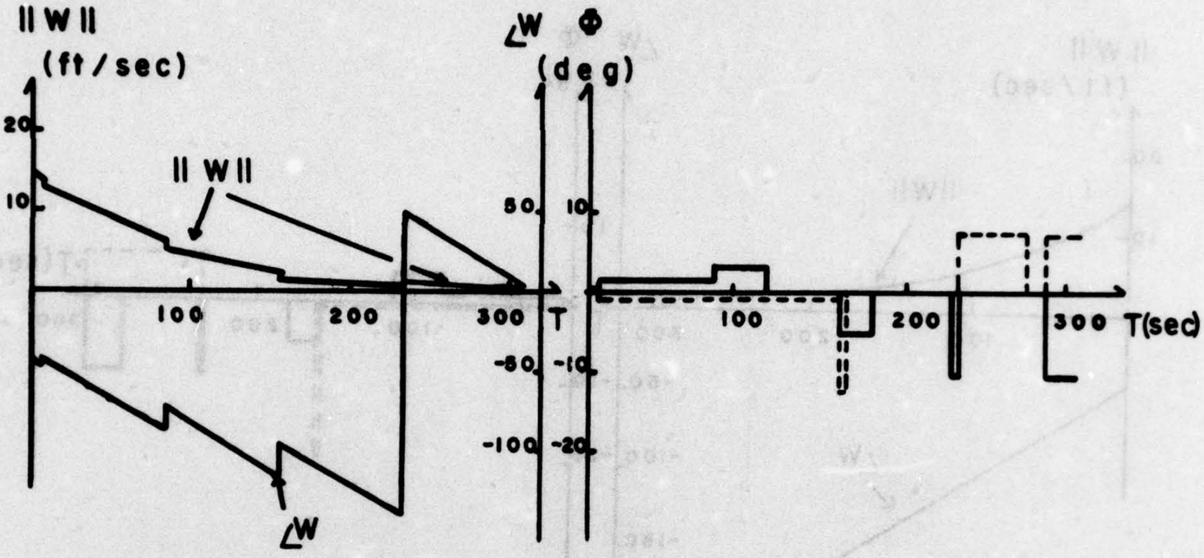


Fig. 10 Simulation Data for Closed-Loop Control, Wind No. 10



Trajs	Initial Data		End Pt. Data	
	$\|P(0)\|$	$\theta(0)$	$\|P(T)\|$	$\Delta\theta(T)$
——	4460	90°	86	136°
- - -	6100	-45°	87	137°

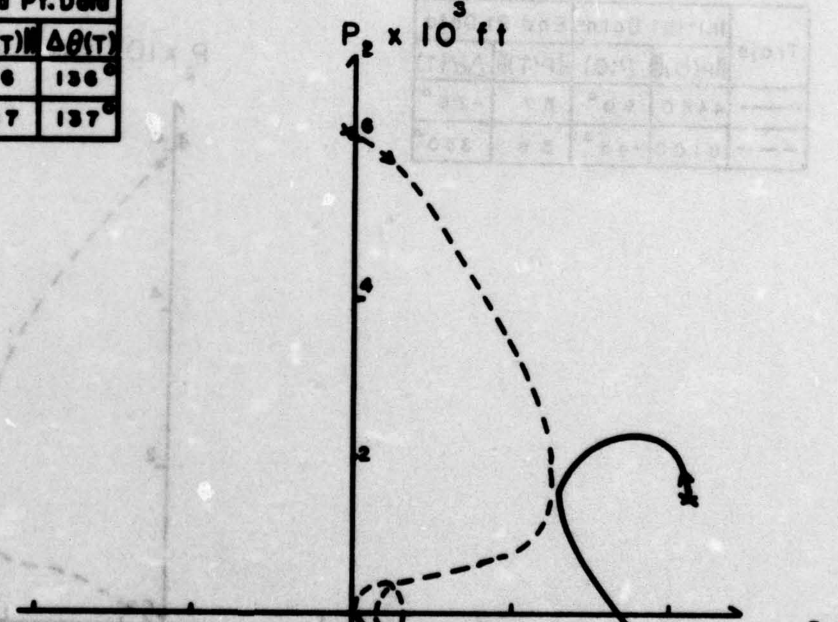


Fig. 11 Simulation Data for Closed-Loop Control, Wind No. 11

References

[1] Pearson, A. E., "Optimal Control of a Gliding Parachute System", Technical Report 73-30-AD, August 1972, USA Natick Laboratories, Natick, MA.

[2] Wei, Kuang-Chung and Pearson, Allan E., "Numerical Solution to the Optimal Control of a Gliding Parachute System", Technical Report 75-107-AMEL, October 1974, USA Natick Laboratories, Natick, MA.

[3] Koopersmith, Robert M. and Pearson, Allan E., "Determination of Trajectories for a Gliding Parachute System", Technical Report 75-117-AMEL, April 1975, U.S. Army Natick Development Center, Natick, MA.

[4] Willsky, A. S. and Lo, J. T-H., "Estimation for Rotational Processes with One Degree of Freedom-Parts I, II and III", IEEE Trans. Automat. Contr., Vol. AC-20, pp. 10-33, February 1975.

[5] Kalman, R. E. and Bucy, R. S., "New Results in Linear Filtering and Prediction Theory", Trans. ASME, J. Basic Engr., Ser. D, pp. 95-108, 1961.

[6] Davison, E. J. and Wong, P. S., "A Robust Algorithm that Minimizes L-Functions in a Finite Number of Steps and Rapidly Minimizes General Functions", Proc. of 1974 IEEE Conf. on Decis. and Contr., Phoenix, AZ, pp. 41-46, November 1974.

APPENDIX

FILE: MAIN   FORTRAN P1   THE BROWN BICENTENNIAL COMPUTER CENTER

```
C     THE PURPOSE IS TO ESTIMATE AND CONTROL A PARACHUTE GLIDING SYSTEM
C     VIA A LEAST SQUARE ESTIMATION SCHEME (ESTM) IN ADDITION TO A BANG-
C     BANG CONTROL SCHEME (PRED).
C     A:PARACHUTE SPEED;CW:COEFF. MATRIX IN DYNAMIC WIND MODEL
C     DTES: EST. INTERVAL LENGTH ; DTIN: INTEGRATION INTERVAL LENGTH
C     ERBD:ERROR BOUND IN RUNGE-KUTTA ROUTINE ;EX: EST.PARACHUTE HEADING
C     EA: EST.X-COMPONENT WIND COEFF. ; EB: EST. Y-COMPONENT WIND COEFF.
C     IM: NO. OF INTEGRATION INTERVAL ; IN: NO. OF EST. INTERVAL
C     IP: EXECUTION CONTROL.IF IP=1,STOP EXECUTION BECAUSE THE GEOMETRIC
C     APPROACH FAILS. ; NC: EST. LOOP COUNT ;TI: INITIAL TIME
C     TF: FINAL TIME ; TB: STORED SWITCH TIME VECTOR ; XIN:INITIAL STATE
C     XF: FINAL STATE VECTOR,AND INTEGRATED VECTOR ; UM:BANG-OFF CONTROL
C     WD: EST. TERMINAL WIND ANGLE ; PI:180 DEGREE IN RADIANS
C     CONV:CONVERSION FACTOR FROM RADIAN TO DEGREE
C     FDS: ESTIMATION INTERVAL ; FDI: INTEGRATION STEP SIZE
C     DG: DEGENERATE BOUND XTF: INTERMEDIATE INITIAL TIME
C     DEV: TERMINAL DEVIATION FROM WIND OPPOSITE
C     TC: INITIAL COUNT TIME ; TGO:FLIGHT TIME IN SECOND
C     RK: FRACTION OF PREDICTION INTERVAL ; ISK:SKIP CONSTANT EST.IF ISK
C     =1
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION XIN(3),XF1(22),TB(2),CW(2,2),WIN(2),EA(2),EB(2),TEX(3)
      COMMON DTES,DTIN,ERBD, TI,TF,NC,IN,IM
      COMMON/F1/A,UM,TB,CW,WIN,UP,DG,TEX,PI,CONV
      COMMON/OUT/TPR,THI,THETA,ENR,ALP,DRBE
      COMMON/PR/WD,IP
      COMMON/EX/TFIN,RK,ISK
      WRITE(6,32)
32    FORMAT(1H ,'DISTURBANCE, NO. OF DROPS')
      READ(5,29)DRBE,NDP
      DO 30 LD=1,NDP
      WRITE(6,28)
28    FORMAT(1H ,'FRACTION OF PRED,',' SKIP CONTROL')
      READ(5,29)RK,ISK
29    FORMAT(D14.8,I2)
      WRITE(6,24)
24    FORMAT(1H ,'INITIAL COUNT TIME , TIME TO GO')
      READ(5,5)TC,TGO
      WRITE(6,11)
11    FORMAT(1H ,'EST.NO.,INTG. NO. , INITIAL STATES, ERROR BOUND')
      READ (5,2)IN,IM,(XIN(I),I=1,3),ERBD
2     FORMAT(2I4,4D14.8)
      WRITE(6,9)
9     FORMAT(1H ,'PARACHUTE SPEED W.R.T. AIR',', INITIAL CONTROL')
      READ(5,5)A,UM
      WRITE(6,10)
10    FORMAT(1H ,'INITIAL COND. OF WIND COMPONENTS')
      READ(5,5)(WIN(I),I=1,2)
5     FORMAT(2D14.8)
      WRITE(6,8)
8     FORMAT(1H ,'COEFF. MATRIX IN DYNAMIC WIND MODEL')
      DO 7 I=1,2
7     READ(5,5)(CW(I,J),J=1,2)
      TFIN=TC+TGO
      PI=DARCOS(-1.D0)
```

```
      CONV=1.8D2/PI
      FDS=TGO/DFLOAT(IN)
      FDI=FDS/DFLOAT(IM)
      DTES=0.1D0*FDS
      DTIN=0.1D0*FDI
      IP=0
C
C INITIALIZE FINAL STATE,INTG. VECTOR,EST. VECTOR
C
      DO 12 I=1,22
12    XF1(I)=0.D0
      EX=0.D0
      DO 13 I=1,2
      EA(I)=0.D0
13    EB(I)=0.D0
      NC=0
      TI=0.D0
      TF=TI+DTES
      TB(2)=TI
      STF=TC
      TB(1)=TF
C
C COMPUTE STATE AND INTEGRATED VECTORS
C
1     CALL PLANT(XIN,XF1)
      IF(NC.EQ.IN)GO TO 22
      DG=DTIN*UM**2+.1D-05
C
C ESTIMATE HEADING AND WIND COEFF.
C
      CALL ESTM(XIN,XF1,EX,EA,EB)
C
C COMPUTE BANG-BANG CONTROL ACCORDING TO MODEL EQ.&EST.STATES.
C
      CALL PRED(XIN,EX,EA,EB,STF,T1M,T2M,UR)
C
C UPDATE INITIAL COND.,START NEXT ESTIMATICN LOOP
C
      NC=NC+1
      IF(NC.EQ.1)GO TO 20
      TI=TI+DTES
      GO TO 21
20    TI=TC
21    TF=TI+FDS
      STF=TF
      UM=UR
      DTES=FDS
      DTIN=FDI
      TB(1)=T1M
      TB(2)=T2M
      GO TO 1
22    WRITE(6,31)THETA
31    FORMAT(1H,'REAL ANG.',2X,D14.8)
      IF(XF1(13).EQ.0.D0)GO TO 25
      DEV=CONV*DMOD((THETA-DATAN2(XF1(14),XF1(13))+PI),(2.D0*PI))
```

```
      GO TO 27
25    IF(XF1(14).GT.0.D0)GO TO 26
      DEV=CONV*DMOD((THETA+1.5D0*PI),(2.D0*PI))
      GO TO 27
26    DEV=CONV*DMOD((THETA+.5D0*PI),(2.D0*PI))
27    WRITE(8,23)DEV
      WRITE(6,23)DEV
23    FORMAT(1H ,'THE DEVIATION FROM THE WIND OPPOSITE IS ',2X,D14.8,2X,
     C'DEGREES.')
30    CONTINUE
      STOP
      END
```

```
      SUBROUTINE PLANT(XIN,YTEL)
C     THE PURPOSE IS WITH GIVEN INITIAL POSITION,HEADING AS WELL AS WIND
C     KNOWN DYNAMICS AND CONTROL LAW,COMPUTE THE CORRESPONDING STATES,AN
C     INTEGRATED VECTOR WHICH IS NEEDED IN LSQ ESTIMATION.
C     INPUT:INITIAL STATE VECTOR 'XIN'
C     OUTPUT:FINAL STATE VECTOR AND SOME INTEGRATED VECTOR 'YTEL'
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION XIN(1),Y(14),DERY(14),PRMT(5),AUX(8,14),YTEL(1),TB(2),
     CCW(2,2),WIN(2)
      COMMON/OUT/TP ,THI,THE,ENR,ALP
      COMMON DS,DI,ED, TI,TF,NC,IN,IM
      COMMON/F1/A,U,TB,CW,WIN,UP
      EXTERNAL FCT1,OUTP1
C
C  INITIALIZE RELATED VECTORS FOR INTEGRATION PURPOSE
C
      PRMT(1)=TI
      PRMT(2)=TF
      PRMT(3)=DI
      PRMT(4)=ED
      TP=TI
      ALP=TI
      NDIM=14
      DO 1 I=1,14
    1 DERY(I)=1.D0/14.D0
      DO 2 I=1,2
    2 Y(I)=XIN(I)
      THI=XIN(3)
      DO 3 I=3,12
    3 Y(I)=0.D0
      DO 6 I=13,14
    6 Y(I)=WIN(I-12)
      WRITE(8,8)
    8 FORMAT(1H ,2X,'TIME',12X,'X(1)',12X,'X(2)',12X,'X(3)',12X,'WIND',
     C12X,'ANGL',10X,'U',14X,'BANK ANG')
      WRITE(6,7)
    7 FORMAT(1H0,2X,'TIME',12X,'X(1)',12X,'X(2)',12X,'X(3)',12X,'WIND',1
     C2X ,'ANGL',10X,'U',14X,'ENERGY')
C
C  START INTEGRATION
C
      CALL DRKGS(PRMT,Y,DERY,NDIM,IHLF,FCT1,OUTP1,AUX)
      WRITE(6,4)IHLF
    4 FORMAT(1H0,'IHLF=',I2)
C
C  STORE FINAL STATE AND INTEGRATED VECTOR
C
      DO 5 I=1,14
    5 YTEL(I)=Y(I)
      RETURN
      END
      SUBROUTINE OUTP1(X,Y,DERY,IHLF,NDIM,PRMT)
      IMPLICIT REAL*8(A-H,O-Z)
```

```
      DIMENSION Y(1),PRMT(1),DERY(1),TB(2),CW(2,2),WIN(2),TEX(3)
      COMMON DS,DI,ED, TI,TF,NC,IN,IM
      COMMON/F1/A,U,TB,CW,WIN,UP,DG,TEX,PI,CV
      COMMCN/OUT/TP,THI,THETA,ENR,ALP
      IF((X.LT.ALP-.5D0*DI).OR.(X.GT.ALP+.5D0*DI))RETURN
      WMAG=DSQRT(Y(13)**2+Y(14)**2)
      IF(Y(14).EQ.0.D0)GO TO 2
      IF(Y(13).EQ.0.D0)GO TO 3
      WANG=DATAN2(Y(14),Y(13))*CV
      GO TO 4
    2 WANG=0.D0
      GO TO 4
    3 WANG=9.0D1
      GO TO 4
    4 BK=DATAN2(A*UP,32.07D0)*CV
      WRITE(8,1)X,Y(1),Y(2),THETA,WMAG,WANG,UP,BK
      ALP=ALP+DFLOAT(IM/10)*DI
      IF((X.LT.TP-.5D0*DI).OR.(X.GT.TP+.5D0*DI))RETURN
C
C PRINT OUT  2  CONSECUTIVE SETS OF TIME,STATES,WIND,CONTROL AND ENERGY
C     EST. INTERVAL
C
      WRITE(6,1)X,Y(1),Y(2),THETA,WMAG,WANG,UP,ENR
    1 FORMAT(1H ,D10.4,7(2X,D14.8))
      TP=TP+DFLOAT(IM )*DI
      RETURN
      END
      SUBROUTINE FCT1(T,Y,DERY)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(1),DERY(1),TB(2),CW(2,2),WIN(2)
      COMMON DS,DI,ED,TI,TF,NC,IN,IM
      COMMON/F1/A,U1,TB,CW,WIN,URE
      COMMON/OUT/TP,TH1,THETA,ENR
      IF((TB(2).LT.TB(1)).AND.(T.GE.TB(1)))GO TO 4
      IF(T.LT.TB(1))GO TO 2
      IF(T.LT.TB(2))GO TO 3
      URE=U1
      UIN=(TB(1)-TI+T-TB(2))*U1
      GO TO 1
    2 UIN=(T-TI)*U1
      URE=U1
       GO TO 1
    3 UIN=(TB(1)-TI)*U1
      URE=0.D0
      GO TO 1
    4 UIN=(TB(1)-TI)*U1
      URE=U1
    1 THETA=TH1+UIN
      DERY(1)=A*DCOS(THETA)+Y(13)
      DERY(2)=A*DSIN(THETA)+Y(14)
      DERY(3)=DSIN(UIN)
      DERY(4)=DCOS(UIN)
      DERY(5)=DERY(1)*DERY(4)
      DERY(6)=DERY(2)*DERY(4)
      DERY(7)=DERY(1)*DERY(3)
```

```
      DERY(8)=DERY(2)*DERY(3)
      DERY(9)=T*DERY(3)
      DERY(10)=T*DERY(4)
      DERY(11)=Y(1)
      DERY(12)=Y(2)
      DERY(13)=CW(1,1)*Y(13)+CW(1,2)*Y(14)
      DERY(14)=CW(2,1)*Y(13)+CW(2,2)*Y(14)
      ENR=U1*UIN
      RETURN
      END
```

```
      SUBROUTINE ESTM(XI,XF,EX,EA,EB)
C     THE PURPOSE IS TO COMPUTE A LEAST SQUARE ESTIMATE OF PARACHUTE
C     HEADING AND COEFFICIENTS OF WIND COMPONENTS.
C     INPUT: INITIAL STATE'XI',FINAL STATE AND INTEGRATED VECTOR 'XF'
C     OUTPUT: ESTIMATED HEADING 'EX',COEFF. VECTORS 'EA'&'EB'.
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION XI(1),XF(1),EA(1),EB(1),P(2,2),CI(2),SI(2),PC(2),PS(2),
     CD(2,2),WIN(2),CW(2,2),TB(2),TEX(5),DERY2(4),Y2(4),PRMT2(5),AUX2(8
     C,4),TEXC(3)
      COMMON DTES,DTIN,ERBD, TI,TF,NC,IN,IM,IPRI
      COMMON/OUT/TPR,THI,THE,ENR,ALP,RAMP
      COMMON/F1/V,U,TB,CW,WIN,UP,DEG,TEXC,PI,CV
      COMMON/F2/TEX
      EXTERNAL FCT2,OUTP2
C
C COMPUTE THE ESTIMATED HEADING 'EX'
C
      P(1,1)=XF(1)-XI(1)
      P(1,2)=TF*XF(1)-TI*XI(1)-XF(11)
      P(2,1)=XF(2)-XI(2)
      P(2,2)=TF*XF(2)-TI*XI(2)-XF(12)
      CI(1)=XF(4)
      CI(2)=XF(10)
      SI(1)=XF(3)
      SI(2)=XF(9)
      PC(1)=XF(5)
      PC(2)=XF(6)
      PS(1)=XF(7)
      PS(2)=XF(8)
      UMRC=XF(6)-XF(7)-(P(2,1)*XF(4)-P(1,1)*XF(3))/DTES
      DMC=XF(5)+XF(8)-(P(1,1)*XF(4)+P(2,1)*XF(3))/DTES
      SD=1.D0/DTES**3
      D(1,1)=4.D0*SD*(TF**2+TF*TI+TI**2)
      D(1,2)=-6.D0*SD*(TF+TI)
      D(2,1)=D(1,2)
      D(2,2)=12.D0*SD
      IF(ENR.LT.DEG)GO TO 24
      RMD=0.D0
      RMU=0.D0
      DO 6 I=1,2
      DO 7 J=1,2
      RMU=RMU+D(I,J)*(P(2,J)*CI(I)-P(1,J)*SI(I))
      RMD=RMD+D(I,J)*(P(1,J)*CI(I)+P(2,J)*SI(I))
7     CONTINUE
6     CONTINUE
      UMER=PC(2)-PS(1)-RMU
      DENOM=PC(1)+PS(2)-RMD
      WRITE(6,27) UMER,DENOM
27    FORMAT(1H ,'NUMERATOR= ',D14.8,2X,'DENOMINATOR= ',D14.8)
      EX =DATAN2(UMER,DENOM)
      WRITE(6,27)UMRC,DMC
      TEXC(1)=DATAN2(UMRC,DMC)
C
```

```
C INITIALIZE EST . COEFF
C
24    DO 2 I=1,2
      EA(I)=0.D0
2     EB(I)=0.D0
C
C COMPUTE ESTIMATED VECTORS 'EA' & 'EB'
C
      DO 3 I=1,2
      DO 4 J=1,2
      EA(I)=EA(I)+D(I,J)*(P(1,J)-V*(DCOS(EX)*CI(J)-DSIN(EX)*SI(J)))
      EB(I)=EB(I)+D(I,J)*(P(2,J)-V*(DSIN(EX)*CI(J)+DCOS(EX)*SI(J)))
4     CONTINUE
3     CONTINUE
C
C COMPUTE CONSTANT EST. VECTOR
C
      TEXC(2)=(XF(1)-XI(1)-V*(XF(4)*DCOS(TEXC(1))-XF(3)*DSIN(TEXC(1))))/
     CDTES
      TEXC(3)=(XF(2)-XI(2)-V*(XF(4)*DSIN(TEXC(1))-XF(3)*DSIN(TEXC(1))))/
     CDTES
C
C STORE LINEAR EST. VECTOR FOR ERROR COMPUTATION.
C
      TEX(1)=EX
      TEX(2)=EA(1)
      TEX(3)=EA(2)
      TEX(4)=EB(1)
      TEX(5)=EB(2)
      DUM=0.D0
C
C COMPUTE THE ESTIMATION ERROR
C
      NDIM2=4
      PRMT2(1)=TI
      PRMT2(2)=TF
      PRMT2(3)=DTIN
      PRMT2(4)=ERBD
      DO 9 I=1,4
9     DERY2(I)=1.D0/4.D0
      DO 10 I=1,2
10    Y2(I)=WIN(I)
      Y2(3)=0.D0
      Y2(4)=0.D0
      IPRI=0
      CALL DRKGS(PRMT2,Y2,DERY2,NDIM2,IHLF2,FCT2,OUTP2,AUX2)
      WRITE(6,15)IHLF2
15    FORMAT(1H ,'IHLF2=',I2)
      IF(Y2(3).LE.1.D-10)IPRI=1
      WRITE(8,5)NC
      WRITE(6,5)NC
5     FORMAT(1H0,'AFTER THE',I4,' TH ESTIMATION, THE ESTIMATED STATE AND
     C ERROR ARE'//1H ,5X,'X(3)',14X,'A(1)',14X,'A(2)',14X,'B(1)',14X,'B
     C(2)',14X,'ERROR')
      WRITE(6,8)EX,EA(1),EA(2),EB(1),EB(2),Y2(3)
```

```
      WRITE(8,8)EX,EA(1),EA(2),EB(1),EB(2),Y2(3)
8     FORMAT(1H ,6(D14.8,4X))
      WRITE(8,8)TEXC(1),TEXC(2),DUM,TEXC(3),DUM,Y2(4)
C
C UPDATE INITIAL CONDITION
C
      DO 16 I=1,2
16    XI(I)=XF(I)
      XI(3)=THE
      DO 28 I=1,2
28    WIN(I)=Y2(I)+RAMP
      EX=EX+THE -THI
      TEXC(1)=TEXC(1)+ THE-THI
      RETURN
      END
      SUBROUTINE FCT2(T,Y2,DERY2 )
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y2(1),WIN(2),DERY2(1),DW(2,2),TX(5),TB(2),TXC(3)
      COMMON DS,DI,ED,TI,TF,NC,IN,IM,IPR
      COMMON/OUT/TPR,THI,THE,ENR
      COMMON/F1/A,U1,TB,DW,WIN,UPD,DGD,TXC,PI,CV
      COMMON/F2/TX
      IF((TB(2).LT.TB(1)).AND.(T.GE.TB(1)))GO TO 3
      IF(T.LT.TB(1))GO TO 2
      IF(T.LT.TB(2))GO TO 3
      UIN=(TB(1)-TI+T-TB(2))*U1
      GO TO 1
2     UIN=(T-TI)*U1
      GO TO 1
3     UIN=(TB(1)-TI)*U1
1     THETA=THI+UIN
      ANGL=TX(1)+UIN
      ANGC=TXC(1)+UIN
      DY1=A*DCOS(THETA)+Y2(1)
      DY2=A*DSIN(THETA)+Y2(2)
      DERY2(1)=DW(1,1)*Y2(1)+DW(1,2)*Y2(2)
      DERY2(2)=DW(2,1)*Y2(1)+DW(2,2)*Y2(2)
      DERY2(3)=(DY1-A*DCOS(ANGL)-TX(2)-TX(3)*T)**2+(DY2
     C-A*DSIN(ANGL)-TX(4)-TX(5)*T)**2
      DERY2(4)=(DY1-A*DCOS(ANGC)-TXC(2))**2+(DY2-A*DSIN(ANGC)-TXC(3)
     C)**2
      RETURN
      END
      SUBROUTINE OUTP2(X,Y2,DERY2,IHLF2,NDIM2,PRMT2)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y2(1),DERY2(1),PRMT2(1)
      RETURN
      END
```

```
      SUBROUTINE PRED(X,EX,EA,EB,T,T1R,T2R,UR)
C     THE PURPOSE IS TO COMPUTE A BANG-BANG CONTROL WHICH DRIVES THE
C     MODEL SYSTEM TO THE TARGET VIA A GEOMETRICAL APPROACH.
C     INPUT: STATE VECTOR 'X',EST.HEADING 'EX',COEFF.'EA' & 'EB',INITIAL
C     TIME.
C     OUTPUT: SWITCH TIMES,CONTROL
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION X(1),EA(1),EB(1),ZIN(2),TBD(2),CWD(2,2),WND(2),TXC(3)
      COMMON DS,DI,ERBD, TI,TF,NP,IN,IM,IPRI
      COMMON/PR/WIND,IMP
      COMMON/F1/V,UDL,TBD,CWD,WND,UPD,DGL,TXC,PI,CV
      COMMON/EX/TFIN,RK,ISK
      COMMON/LAST/VT,OLJ
C
C COORDINATE TRANSFORMATION
C
      TEND=1.D0/DFLOAT(IN-NP)
      SCAL=TFIN-T
      WRITE(8,14)EA(1),EA(2),EB(1),EB(2)
14    FORMAT(1H ,4(D14.8,2X))
      VT=V*SCAL
      ZIN(1)=X(1)/VT+(EA(1)+EA(2)*(TFIN+T)/2.D0)/V
      ZIN(2)=X(2)/VT+(EB(1)+EB(2)*(TFIN+T)/2.D0)/V
      WIND=DATAN2((EB(1)+EB(2)*TFIN),(EA(1)+EA(2)*TFIN))-PI
      RHO=DSQRT(ZIN(1)**2+ZIN(2)**2)
      PHI=DATAN2(ZIN(2),ZIN(1))
      WRITE(8,9)RHO,PHI,WIND
9     FORMAT(1H ,8X,'RHO',12X,'PHI',12X,'WIND ANGLE'/1H ,2X,4(D14.8,2X))
      T1S=0.D0
      T2S=0.D0
      IMP=0
      PS1=0.D0
      PS2=0.D0
      PSU=0.D0
      POLJ=1.D10
      PEFF=0.D0
      DEFF=0.D0
      OEFF=0.D0
      IF(RHO.GE.1.D0)IMP=1
C
C COMPUTE BANG-BANG CONTROL ACCORDING TO LINEAR WIND MODEL
C
      CALL MGEO(RHO,PHI,EX,T1S,T2S,U)
      IF(IMP.NE.1)GO TO 6
      DET=VT*DSQRT(OLJ)
      IMP=0
      IF(TEND.GE.T2S)DEFF=DABS((TEND-T2S)*U)
      CPS1=T1S
      CPS2=T2S
      CPU=U
      IF(IPRI.EQ.1)GO TO 11
      IF(ISK.EQ.1)GO TO 12
C
```

```
C COMPUTE BANG-BANG CONTROL ACCORDING TO CONSTANT WIND MODEL
C
      ZIN(1)=(X(1)+TXC(2)*SCAL)/VT
      ZIN(2)=(X(2)+TXC(3)*SCAL)/VT
      WIND=DATAN2(TXC(3),TXC(2))-PI
      RHO=DSQRT(ZIN(1)**2+ZIN(2)**2)
      PHI=DATAN2(ZIN(2),ZIN(1))
      WRITE(8,9)RHO,PHI,WIND
      IF(RHO.GE.1.D0)IMP=1
      SP=TXC(1)
      CALL MGEO(RHO,PHI,SP,T1S,T2S,U)
      IF(IMP.NE.1)GO TO 6
      IMP=0
      POLJ=VT*DSQRT(OLJ)
      PS1=T1S
      PS2=T2S
      PSU=U
C
C COMPUTE BANG-BANG CONTROL ACCORDING TO LINEAR PLUS CONSTANT WIND MODEL
C
12    DSCA=SCAL*RK
      RSCA=DSCA+T
      REM=SCAL-DSCA
      ZIN(1)=(X(1)+(EA(1)+EA(2)*(DSCA+2.D0*T)/2.D0)*DSCA+(EA(1)+EA(2)*
     CRSCA)*REM)/VT
      ZIN(2)=(X(2)+(EB(1)+EB(2)*(DSCA+2.D0*T)/2.D0)*DSCA+(EB(1)+EB(2)*
     CRSCA)*REM)/VT
      WIND=DATAN2(EB(1)+EB(2)*RSCA,EA(1)+EA(2)*RSCA)-PI
      RHO=DSQRT(ZIN(1)**2+ZIN(2)**2)
      PHI=DATAN2(ZIN(2),ZIN(1))
      WRITE(8,9)RHO,PHI,WIND
      IF(RHO.GE.1.D0)IMP=1
      CALL MGEO(RHO,PHI,EX,T1S,T2S,U)
      IF(IMP.NE.1)GO TO 6
      OLJ=VT*DSQRT(OLJ)
      IF(T1S.GT.0.D0)OEFF=DABS(DMIN1(T1S,TEND)*U)
      IF(TEND.GT.T2S)OEFF=DABS((TEND-T2S)*U)
      WRITE(8,8)
8     FORMAT(1H ,8X,'T1',15X,'T2',8X,'CONTROL',8X,'EXP MISS DISTANCE')
      WRITE(8,7)CPS1,CPS2,CPU,DET,PS1,PS2,PSU,POLJ,T1S,T2S,U,OLJ
7     FORMAT(1H ,2X,4(D14.8,2X)/1H ,2X,4(D14.8,2X)/1H ,2X,4(D14.8,2X))
      IF(CPS1.EQ.0.D0)GO TO 13
      IF(T1S.EQ.0.D0)GO TO 11
      DEFF=DABS(DMIN1(CPS1,TEND)*CPU)
13    IF((OEFF.GE.DEFF).AND.(OEFF.GE.PEFF))GO TO 4
      IF((PEFF.GE.OEFF).AND.(PEFF.GE.DEFF))GO TO 10
11    T1S=CPS1
      T2S=CPS2
      U=CPU
      GO TO 4
10    T1S=PS1
      T2S=PS2
      U=PSU
      GO TO 4
C
```

```
C COMPUTE SWITCH TIMES,CONTROL IN INERTIAL COORDINATE
C
6     T1R=T1S*SCAL+T
      T2R=T2S*SCAL+T
      UR=U/SCAL
      WRITE(8,2)
      WRITE(6,2)
2     FORMAT(1H0,'THE SWITCH TIMES AND CONTROL ARE '//1H ,5X,'T1',15X,'T
     C2',15X,'U'/)
      WRITE(8,1)T1R,T2R,UR
      WRITE(6,1)T1R,T2R,UR
1     FORMAT(1H ,3(D14.8,2X))
      GO TO 3
4     WRITE(6,5)
5     FORMAT(1H0,'THE FIRST GEOMETRICAL APPROACH FAILS')
      GO TO 6
   3  RETURN
      END
```

```
      SUBROUTINE MGEO(RHO,PHI,THETA,T1S,T2S,USTAR)
C     THIS ROUTINE COMPUTES A BANG-OFF-BANG CONTROL VIA A GEOMETRICAL
C     APPRCACH. THIS IS POSSIBLE ONLY WHEN THE PARACHUTE IS WITHIN THE
C     UNIT CIRCLE. FOR THE CASE WHEN IT FALLS OUTSIDE THE UNIT CIRCLE,
C     A SECOND APPROACH IS USED TO COMPUTE A BANG-OFF, OFF-BANG, BANG,
C     OR OFF CONTROL DEPENDING UN WHICH WOULD RESULT A MINIMAL EXPECTED
C     MISS DISTANCE.
      IMPLICIT REAL * 8 (A-H,O-Z)
      DIMENSION R(2),E(2),G(2),F(2),TSTAR(2),TSTARK(2,20),T1(2,20),T2(2,
     C20),U(2),UM(2),DIST(2)
      COMMCN/LAST/VT,OLJ
      COMMON/PR/SBETA,IMP
      TWOPI=2.DO*DARCOS(-1.DO)
      WRITE(8,3)
      N1=5
C
C  INITIALIZE BEST CONTROL,ENERGY,BANG-OFF TIMES.
C
      USTAR=0.DO
      BETA=SBETA
      ESTAR=1.D10
      T1S=0.DO
      T2S=0.DO
      ID=0
      IF(IMP.EQ.1)GO TO 119
      DO 19 NN=1,N1
      N=NN-(1+N1)/2
      FN=DFLOAT(N)
      PSIN=TWOPI*FN-THETA+BETA
      IF(PSIN.EQ.0.DO) GO TO 19
      IF(DSIN(THETA)-PSIN*RHO*DCOS(PHI).NE.0.DO) GO TO 7
      IF(1.DO-DCOS(THETA)-PSIN*RHO*DSIN(PHI).NE.0.DO) GO TO 7
      IF(IABS(N).GE.2) GO TO 19
      R(1)=1.DO/PSIN
      U(1)=PSIN
      E(1)=PSIN**2
C
C UPDATE BEST CONTROL,ENERGY.
C
      IF(ESTAR.LT.E(1))GO TO 19
      IF(ESTAR.EQ.E(1))GO TO 103
      ESTAR=E(1)
      USTAR=U(1)
      ID=1
      GO TO 99
103   IF(DABS(U(1)).GE.DABS(USTAR))GO TO 19
      USTAR=U(1)
      ID=1
99    WRITE(8,98)
98    FORMAT(1H ,'T1=T2',//1H ,5X,'N',7X,'PSIN',17X,'R(1)',10X,'U(=)',10
     1X,'E(1)')
      WRITE(8,25) N,PSIN,R(1),U(1),E(1)
25    FORMAT(1H ,5X,I2,5X,4(D14.8,5X))
      GO TO 19
C
```

```
C     COMPUTE THE TURN RADIUS
C
    7 A=PSIN**2-2.D0*(1.D0-DCOS(THETA-BETA))
      B=PSIN+RHO*(DSIN(PHI-THETA)-DSIN(PHI-BETA))
      C=1.D0-RHO**2
      D=B**2-A*C
      IF(D.LT.0.D0) GO TO 19
      R(1)=(B+DSQRT(D))/A
      R(2)=(B-DSQRT(D))/A
      IF(D.NE.0.D0) GO TO 15
      J=1
      GO TO 8
   15 J1=0
      J2=0
      IF(R(1)*PSIN.LE.0.D0) GO TO 5
      IF(R(1)*PSIN.GE.1.D0) GO TO 5
      J1=1
    5 IF(R(2)*PSIN.LE.0.D0) GO TO 6
      IF(R(2)*PSIN.GE.1.D0) GO TO 6
      J2=1
    6 J=J1+J2
      IF(J.EQ.0) GO TO 19
      IF(J.EQ.2) GO TO 8
      IF(J1.EQ.1) GO TO 8
      R(1)=R(2)
C
C     COMPUTE THE CONTROL, ENERGY, AND TURN ANGLE
C
    8 DO 9 I=1,J
      U(I)=1.D0/R(I)
      E(I)=PSIN/R(I)
      F(I)=(R(I)*(DSIN(THETA)-DSIN(BETA))-RHO*DCOS(PHI))/(1.D0-R(I)*PSIN
     C)
      G(I)=(R(I)*(DCOS(BETA)-DCOS(THETA))-RHO*DSIN(PHI))/(1.D0-R(I)*PSIN
     C)
      IF(F(I).EQ.-1.D0) GO TO 10
      TSTAR(I)=DSIGN(1.D0,G(I))*DARCOS(F(I))
      GO TO 11
   10 TSTAR(I)=DARCOS(F(I))
   11 K1=IABS(N)+1
C
C     COMPUTE THE SWITCH TIMES
C
      DO 12 KK=1,K1
      IF(R(I)) 16,16,18
   16 K=KK-1+N
      GO TO 20
   18 K=KK-1
   20 TSTARK(I,K)=TSTAR(I)+TWOPI*FLOAT(K)
      T1(I,K)=R(I)*(TSTARK(I,K)-THETA)
      T2(I,K)=1.D0-R(I)*(TWOPI*FN-TSTARK(I,K)+BETA)
      IF(T1(I,K).LT.0.D0) GO TO 12
      IF(T2(I,K).GT.1.D0) GO TO 12
   17 WRITE(8,4) N,PSIN,R(I),T1(I,K),T2(I,K),TSTARK(I,K),U(I),E(I)
C
```

```
C UPDATE BEST CONTROL,ENERGY,TIMES.
C
      IF(ESTAR.LT.E(I))GO TO 12
      IF(ESTAR.EQ.E(I))GO TO 100
      ESTAR=E(I)
      USTAR=U(I)
      T1S=T1(I,K)
      T2S=T2(I,K)
      ID=2
      GO TO 12
100   IF(DABS(U(I)).GE.DABS(USTAR))GO TO 12
      USTAR=U(I)
      T1S=T1(I,K)
      T2S=T2(I,K)
      ID=2
12    CONTINUE
9     CONTINUE
19    CONTINUE
      IF(ID.EQ.0)GO TO 108
      IF(ID.NE.1)GO TO 101
C
C PRINT OUT THE BEST CONTROL,ENERGY,TIMES.
C
      WRITE(8,106)
106   FORMAT(1H0,20X,'BEST CONTROL',3X,'MIN ENERGY',/)
      WRITE(8,105)USTAR,ESTAR
105   FORMAT(1H ,5X,'T1=T2',10X,2(D14.8))
      GO TO 102
101   WRITE(8,107)
107   FORMAT(1H0,8X,'T1',15X,'T2',8X,'BEST CONTROL',5X,'MIN ENERGY')
      WRITE(8,104)T1S,T2S,USTAR,ESTAR
104   FORMAT(1H ,2X,4(D14.8,2X))
      GO TO 102
108   IMP=1
C
C THE FIRST GEOMETRICAL APPROACH FAILS IF ID=0.
C
      WRITE(8,109)
109   FORMAT(1H0,'NO FEASIBLE BANG-OFF-BANG CONTROL EXISTS.')
C
C START THE SECOND GEOMETRICA  APPROACH
C
119   X1=RHO*DCOS(PHI)
      X2=RHO*DSIN(PHI)
      OLJ=1.D20
      OSW1=0.D0
      OSW2=0.D0
      DO 115 N=1,5
      BETA=SBETA+TWOPI*DFLOAT(N-3)
      IF(THETA.EQ.BETA)GO TO 115
C
C COMPUTE SINGLE SWITCH TIME AND CONTROL
C
      DCX=DCOS(THETA)
      DSX=DSIN(THETA)
```

```
      DCB=DCOS(BETA)
      DSB=DSIN(BETA)
      SBX=DSB-DSX
      CBX=DCB-DCX
      DIF=THETA-BETA
      DENOM=(1.D0-(DSIN(DIF)/DIF)**2+4.D0*((DSIN(DIF/2.D0))**4)/DIF**2
      DIST(1)=((X2*DCB-X1*DSB+(X1*CBX+X2*SBX+1.D0-DCOS(DIF))/DIF)**2)/DE
     CNOM
      DIST(2)=((X1*DSX-X2*DCX-(X1*CBX+X2*SBX-1.D0+DCOS(DIF))/DIF)**2)/DE
     CNOM
      UM(1)=1.D0+X1*DCB+X2*DSB+(X1*SBX-X2*CBX-DSIN(DIF))/DIF
      UM(2)=-X1*DCX-X2*DSX-(X1*SBX-X2*CBX+DSIN(DIF)-(2.D0-2.D0*DCOS(DIF)
     C)/DIF)/DIF
      DO 111 K=1,2
      IF((UM(K).GT.0.D0).AND.(UM(K).LE.DENOM))GO TO 117
C
C MINIMAL MISS DISTANCE OCCURS AT BOUNDARY POINT
C
      IF((.NOT.((UM(K).LE.0.D0).OR.(K.EQ.1))).OR.((UM(K).LE.0.D0).AND.(K
     C.EQ.1)))GO TO 118
      SWT1=1.D0
      SWT2=0.D0
      UPJ=(X1-SBX/DIF)**2+(X2+CBX/DIF)**2
      UB=-DIF
      GO TO 113
118   SWT1=0.D0
      SWT2=1.D0
      UPJ=(X1+DCB)**2+(X2+DSB)**2
      UB=0.D0
      GO TC 113
C
C MINIMAL MISS DISTANCE OCCURS AT SWITCH TIME
C
117   UPJ=DIST(K)
      IF(K.EQ.1)GO TO 116
C
C OFF-BANG CONTROL AND SWITCH TIME ARE COMPUTED
C
      SWT1=0.D0
      SWT2=UM(2)/DENOM
      UB=-DIF/(1.D0-SWT2)
      GO TO 113
C
C BANG-OFF CONTROL AND SWITCH TIME ARE COMPUTED
C
116   SWT1=UM(1)/DENOM
      SWT2=1.D0
      UB=-DIF/SWT1
113   RDIST=VT*DSQRT(UPJ)
      IF(UPJ.GE.OLJ)GO TO 111
      OLJ=UPJ
      OSW1=SWT1
      OSW2=SWT2
      OU=UB
111   CONTINUE
```

```
115   CONTINUE
      T1S=OSW1
      T2S=OSW2
      USTAR=OU
    3 FORMAT(1H0,6X,'N',9X,'PSI N',11X,'R',13X,'T1',12X,'T2',6X, 'THETA
     CSTAR K',7X,'U',13X,'E',/)
    4 FORMAT(1H ,5X,I2,1X,7(D14.8,1X))
102   RETURN
      END
```